The University of the State of New York

263D HIGH SCHOOL EXAMINATION

ADVANCED ALGEBRA

Thursday, June 20, 1935 — 9.15 a. m. to 12.15 p. m., only

Instructions

Do not open this sheet until the signal is given.

Group I

This group is to be done first and the maximum time allowed for it is one and one half hours.

If you finish group I before the signal to stop is given you may begin group II. However, it is advisable to look your work over carefully before proceeding, since *no credit will be given any answer in group I which is not correct and in its simplest form.*

When the signal to stop is given at the close of the one and one half hour period, work on group I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

Group II

Write at top of first page of answer paper to group II (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in advanced algebra.

The minimum time requirement is five recitations a week for half a school year after the completion of intermediate algebra.

The use of the slide rule will be allowed for checking but all computations with tables must be shown on the answer paper.

Fill in the following lines:

Name of school.....Name of pupil.....

Detach this sheet and hand it in at the close of the one and one half hour period.

Group I

Answer all questions in this group. Each correct answer will receive 2½ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1 One of the roots of the equation $x^4 - 3x^2 + 6x - 4 = 0$ is 1; what is the product of the other three roots? *Ans*.....

2 What is the only possible rational fractional root of the equation $3x^3 + 4x^2 + 4x + 1 = 0$? *Ans*.....

3 Write as a single term the sum of $\log_2 8 + \log_{10} 10 + \log_5 1$ *Ans*.....

4 Express $\frac{2+i}{2-i}$ in the form $a + bi$ *Ans*.....

5 Write in simplest form the *third* term of the expansion

$$(x^2 - \frac{1}{x^2})^5$$ *Ans*.....

6 If the complex number $\sqrt{3} + i$ is represented graphically by the point P and a line is drawn from P to the origin, how many degrees are there in the angle formed by this line and the positive portion of the axis of real numbers? *Ans*.....

7 Given $a^x = b^y$; express $\frac{x}{y}$ as the ratio between two logarithms. *Ans*.....

8 Write the equation of a line parallel to the line $y = \frac{2x}{3} + 15$ and having the same y-intercept as the line $y = 2x - 3$ *Ans*.....

9 Transform the equation $x^4 + 3x - 2 = 0$ into an equation whose roots are twice the roots of the given equation. *Ans*.....

10 In how many points do the graphs of $y^2 = 4x$ and $x^2 = 4y$ intersect? *Ans*.....

11 The equation $x^6 + 4x^4 + 3x^2 + 16 = 0$ has (*a*) 4 imaginary and 2 real roots, (*b*) 4 real and 2 imaginary roots, (*c*) 6 imaginary roots or (*d*) 6 real roots. Which is correct, (*a*), (*b*), (*c*) or (*d*)? *Ans*.....

12 Transform the equation $x^4 - 3x^2 + 2 = 0$ into an equation whose roots are greater by 1 than the roots of the given equation. *Ans*.....

13 Express t in terms of a and s, given $s = t^2 + at + 1$ *Ans*.....

14 How many different amounts of money can be made with a quarter, a dime, two nickels and a penny, if two coins are taken at a time? *Ans*.....

15 In tossing three coins, what is the probability of getting two heads and a tail? *Ans*.....

Directions (questions 16–19) — Indicate whether each of the following statements is *always* true, *sometimes* true or *never* true by writing the word *always, sometimes* or *never* on the dotted line at the right.

[The equations in questions 16–19 are rational and integral with real and rational coefficients.]

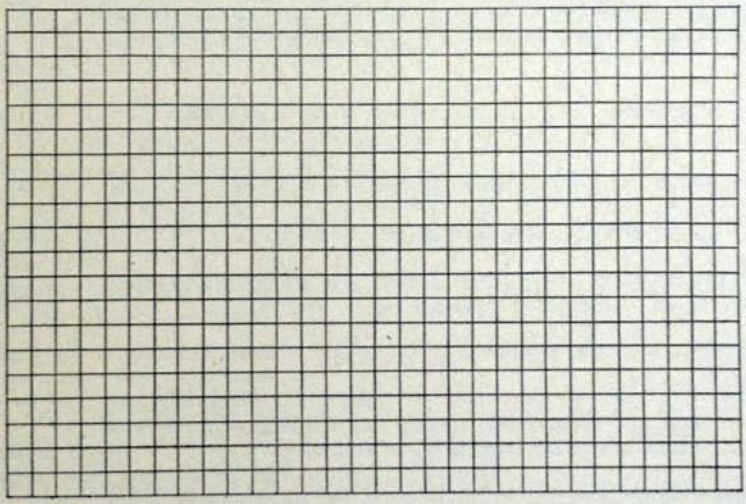
16 The sum of the roots of $a_0 x^n + a_1 x^{n-1} + \ldots + a_{n-1} x + a_n = 0$ is $-\frac{a_1}{a_0}$ *Ans*.....

17 The equation $a_0 x^n + a_1 x^{n-1} + a_2 x^{n-2} + \ldots + a_{n-1} x + a_n = 0$, where n is an odd number, contains at least one real root. *Ans*.....

18 The graph of $f(x) = a_0 x^n + a_1 x^{n-1} + a_2 x^{n-2} + \ldots + a_{n-1} x + a_n$ crosses the x-axis in $n + 1$ points. *Ans*.....

19 An equation of the form $a_0 x^n + a_1 x^{n-1} + \ldots + a_{n-1} x + a_n = 0$, where $a_0 > 1$, has no integral roots. *Ans*.....

20 On the diagram below sketch the graph of the function $y = \log x$, using the following values of x: .1, 1, 2, 5, 10.



182

See instructions for group II on page 1.

Group II

Answer five questions from this group. Full credit will not be granted unless all operations (except mental ones) necessary to find results are given; simply indicating the operations is not sufficient. Each answer should be reduced to its simplest form. Purely arithmetical solutions for problems will not be accepted.

21 Find the real root of $x^3 - 2x - 5 = 0$ correct to the *nearest tenth*. [10]

22 *a* For what value of k is the function $2x^4 - 3x^3 + kx^2 - 7x + 3$ exactly divisible by $x - 3$? [5]

b Find the two integral roots of $x^3 - 9x^{\frac{3}{2}} + 8 = 0$ [5]

23 The time of vibration of a simple pendulum is given by the formula $t = 2\pi\sqrt{\frac{l}{g}}$, where t is the time in seconds, l the length of the pendulum in feet and g the gravitational constant. Given $g = 32.16$ feet per second per second and $\pi = 3.142$, find, correct to the *nearest hundredth of a foot*, the length of a pendulum whose time of vibration is 2.75 seconds. [Use logarithms.] [10]

24 An open box is made from a rectangular sheet of cardboard 18″ x 6″ by cutting equal squares from the corners and turning up the sides. The capacity of the box thus formed is 56 cubic inches. Find a side of one of the squares cut out. [10]

25 *a* Prove that the number of combinations of n things taken r at a time is equal to the number of combinations of n things taken $n - r$ at a time; that is, ${}_nC_r = {}_nC_{n-r}$ [8]

b Find the value of ${}_{100}C_{98}$ [2]

26 Sand and gravel have been mixed in two separate piles. In the first pile the sand is mixed with the gravel in the ratio 1:3 and in the second in the ratio 3:5. What amount must be taken from each pile to form a third pile which shall contain 5 cubic yards of sand and 9 cubic yards of gravel? [10]

27 Given the equation $x^2 - kx + k = 0$

a Express the discriminant d as a function of k. [1]

b Plot the graph of d expressed as a function of k. [6]

c From the graph made in answer to *b*, determine the values of k that will make the roots of the original equation (1) real and equal, (2) real and unequal, (3) complex conjugates. [3]

*28 *a* The velocity of a moving body is given by the equation $V = 3(1 + t)(5 - t)$. For what value of t is the velocity greatest? [4]

b Find the slope of the line tangent to the graph of the function $y = \sqrt{x^2 - 8}$ at the point on the graph whose abscissa is + 3. [6]

*29 *a* Express in polar form *one* imaginary root of the equation $x^3 - 1 = 0$ [7]

b Write *each* of the following numbers in polar form:

(1) $1 + i\sqrt{3}$, (2) $-2i$, (3) -2 [3]

* This question is based on one of the optional topics in the syllabus.