The University of the State of New York

257TH HIGH SCHOOL EXAMINATION

ADVANCED ALGEBRA

Thursday, June 22, 1933 — 9.15 a. m. to 12.15 p. m., only

Instructions

Do not open this sheet until the signal is given.

Answer all questions in part I and five questions from part II.

Part I is to be done first and the maximum time to be allowed for this part is one and one half hours. Merely place the answer to each question in the space provided; no work need be shown.

If you finish part I before the signal to stop is given you may begin part II. However, it is advisable to look your work over carefully before proceeding to part II, since *no credit will be given any answer in part I which is not correct and reduced to its simplest form.*

When the signal to stop is given at the close of the one and one half hour period, work on part I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

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Fill in the following lines:

Name of school.....Name of pupil.....

Detach this sheet and hand it in at the close of the one and one half hour period.

Part I

Answer all questions in this part. Each question has 2½ credits assigned to it; no partial credit should be allowed. Each answer must be reduced to its simplest form.

1 Express with a real denominator the sum of the fractions $\frac{2}{1-i}$ and $\frac{1}{i-1}$ Ans.....

2 By means of logarithms find to the *nearest tenth* the real root of the equation $x^3 = 35$ Ans.....

3 Write the first *three* terms of the expansion $(x - \frac{1}{x})^6$ Ans.....

4 Transform the equation $2x^2 - 7 = 0$ into an equation with integral coefficients, that of the highest-degree term being unity. Ans.....

5 Find the rational fractional root of the equation $2x^3 + x^2 + 9x - 5 = 0$ Ans.....

6 Given the equation $2x^3 - x^2 - 8x + 4 = 0$, in which two of the roots are equal in absolute value but opposite in sign; find the third root. Ans.....

7 In how many ways can a bowling team of 4 be chosen from a club of 12 men? Ans.....

8 There are 5 trails leading to the top of a mountain. In how many ways may one go to the top and return either by the same or by a different trail? Ans.....

9 Given $\log_b A^c = N$; to what power must b be raised to equal A? Ans.....

10 The numbers $1 + i\sqrt{3}$ and $2i$ are represented graphically by the points P and Q respectively and the origin is represented by the point O. How many degrees are there in the angle POQ? Ans.....

11 What is the equation of the straight line passing through the origin and parallel to the straight line whose equation is $x + 2y + 4 = 0$? Ans.....

12 If a ball is thrown vertically into the air with an initial velocity V_0, the height S to which it will rise after any number of seconds t is given by the formula $S = V_0 t - 16t^2$. Express t as a function of S and V_0. Ans.....

13 What is the maximum number of points in which the graph of the equation $xy = 4$ can cut the graph of the equation $x^2 + y^2 = K$, if K is a positive number? Ans.....

[OVER]

14 The mathematics involved in dividing an octave into 12 approximately equal tone intervals is that of inserting 11 geometric means between the numbers 1 and 2. Express in radical form the common ratio. *Ans*.....

Directions (questions 15–19) — Some of these statements are *always* true, others are *sometimes* true and the rest are *never* true. Indicate your opinion in each case by writing the word *always*, *sometimes* or *never* on the dotted line at the right.

15 The product of two complex conjugate numbers is a real and positive number. *Ans*.....

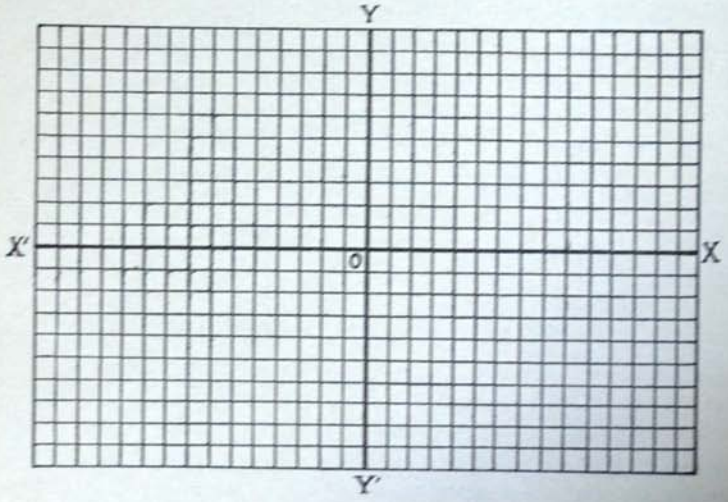
16 The number of positive roots of a rational integral equation is exactly equal to the number of variations of signs in that equation. *Ans*.....

17 In an arithmetic series of n terms S represents the sum, d the difference and a the first term; if S and d are variables and a and n constants, S is a linear function of d. *Ans*.....

18 The graph of the function $y = a_0 x^n + a_1 x^{n-1} \ldots a_n$, in which the coefficients a_0, a_1, etc. are real and rational and a_0 is not equal to zero, cuts the x-axis in an even number of points if n is odd. *Ans*.....

19 The graph of the equation $ax^2 + by^2 = c$ is an ellipse. *Ans*.....

20 On the diagram below draw the graph of the function $y = 2^x$, using values of x from -2 to $+2$ inclusive.



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Write at top of first page of answer paper to part II (a) name of school where you have studied, (b) number of weeks and recitations a week in advanced algebra.
The minimum time requirement is five recitations a week for half a school year after the completion of intermediate algebra.

Part II

Answer five questions from this part. Full credit will not be granted unless all operations (except mental ones) necessary to find results are given; simply indicating the operations is not sufficient. Each answer should be reduced to its simplest form. Purely arithmetical solutions for problems will not be accepted.

In the examination in advanced algebra the use of the slide rule will be allowed for checking, provided all computations with tables are shown on the answer paper.

21 One root of the equation $x^4 - 6x^3 + 9x^2 - 6x - 2 = 0$ is $1 - i$; find the other roots. [10]

22 In finding the maximum deflection of a beam of given length, loaded in a certain way, it is necessary to solve the equation $4x^3 - 15x^2 + 15x - 2 = 0$. Find to the nearest tenth the root of the equation lying between 2 and 3. [10]

23 A man walked 9 miles at a certain rate and then walked 5 miles farther at a rate $\frac{1}{2}$ mile an hour slower. If he had walked the entire distance at the slower rate, it would have taken him 15 minutes longer. Find the two rates. [7, 3]

24 The time t in hours required, under favorable conditions, for 1000 bacteria in a culture to increase to 6000 is given by the formula $6000 = 1000e^{nt}$ in which $e = 2.718$ and $n = .44$. Find to the nearest tenth the value of t. [10]

25 Prove that if $f(x)$ is divided by $x - c$, the remainder is $f(c)$. [10]

26 A machine can do a piece of work in 22 hours less time than is required by one man to do the same work by hand. The work is started on the machine, but after 45 minutes it is found necessary to finish it by hand. Five men, all of whom work at the same rate, are put on the job and they finish it by hand in 3 hours. How many hours are required for doing the entire work by machine? [7, 3]

27 *a* Using the same set of axes, plot the graph of (1) $xy = 4$ from $x = -4$ to $+4$ inclusive, (2) $x = y^2 - 1$ from $x = -1$ to $+4$ inclusive. [8]

b From the graphs made in answer to *a* find to the nearest tenth a real solution of the following set of equations:

$$xy = 4$$
$$x = y^2 - 1 \quad [2]$$

*28 A rectangular chicken yard is to be made from 60 feet of chicken wire, the side of the chicken coop being used for one side of the yard. Let y represent the length of the side parallel to the coop and let x represent the length of each end of the yard.

a Express the length of the fence as a function of x and y. [2]

b Express the area (A) of the yard as a function of x. [2]

c Find the rate at which A is changing with respect to x. [5]

d Find the dimensions of the yard, if the area is to be a maximum. [5]

*29 In each of the following functions find the rate at which y is changing with respect to x:

$$y = x + \frac{1}{x} \quad [5]$$

$$y = \sqrt{x^2 + 8x} \quad [5]$$

*30 Express in polar form the real root and one of the imaginary roots of the equation $x^4 + 8 = 0$ [5, 5]

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*This question is based on one of the optional topics in the syllabus.