

ADVANCED ALGEBRA

Thursday, June 16, 1927 — 9.15 a. m. to 12.15 p. m., only

Write at top of first page of answer paper (a) name of school where you have studied, (b) number of weeks and recitations a week in (1) elementary algebra, (2) intermediate algebra, (3) advanced algebra.

The minimum time requirement is five recitations a week in algebra for two school years.

Answer eight questions. Each answer should be reduced to its simplest form.

In the examination in advanced algebra the use of the slide rule will be allowed for checking, provided all computations with tables are shown on the answer paper.

1 The sum of three numbers in arithmetic progression is 36. If the first number is multiplied by 2, the second by 3 and the third by 6, the resulting numbers will be in geometric progression. Find the three numbers. $[12\frac{1}{2}]$

2 Given the equation $x^3 - 3x^2 - x + 5 = 0$

a Transform this equation into an equation whose roots are 2 less than the roots of the given equation. [6]

b Transform the given equation into an equation whose roots are twice the roots of the given equation. $[6\frac{1}{2}]$

3 In the formula $P = P_0 e^{kt}$, find the value of k to four figures when $P_0 = 14.72$, $e = 2.718$ and $P = 6.480$. $[12\frac{1}{2}]$

4 The equation $2x^3 + 3x^2 + 4x + 12 = 0$ has one integral root; find the three roots of the equation. $[12\frac{1}{2}]$

5 When a certain number is divided by the product of its two digits, the quotient is 3 and when 18 is added to the number, its digits are reversed; find the number. $[5, 7\frac{1}{2}]$

6 a Expand by the binomial theorem $(x^{-1} - y^3)^4$ [8]

b Without expanding find the middle term of $(x - \frac{1}{x})^8$ $[4\frac{1}{2}]$

7 a Show that $x^6 + 2x^3 + 3x - 4 = 0$ has exactly four complex (imaginary) roots. $[4\frac{1}{2}]$

b In the equation $x^3 - 8x^2 + 5x + k = 0$, find the values of k that make two roots equal. [8]

8 a There are 10 points in a plane, no three of which lie in the same straight line. If these points are joined by straight lines, how many triangles are formed? [6]

b A collector has 5 paintings, of which 2 are landscapes and 3 portraits. In how many ways can he hang them all in a horizontal line on one wall so that a landscape is always at the extreme left? $[6\frac{1}{2}]$

9 a Write in the form $a + bi$: $(2 + i)^2$; -3 ; $\frac{4 + \sqrt{-9}}{3}$ $[2, 2, 2]$

b Represent $5 + 4i$ and $4 - 3i$ graphically; find graphically $(5 + 4i) - (4 - 3i)$ $[1, 1, 4\frac{1}{2}]$

10 By Horner's method find to the nearest hundredth the real root of $x^3 - x^2 - 6 = 0$ $[12\frac{1}{2}]$

11 a Plot the graph of $y = x(x + 1)(x - 2)$ from $x = -2$ to $x = 3$ inclusive. $[6\frac{1}{2}]$

b Plot the graph of $y = 4x$. [3]

c From these graphs determine the solutions common to these two equations. [3]