The University of the State of New York

233D HIGH SCHOOL EXAMINATION

ADVANCED ALGEBRA

Monday, June 15, 1925 — 9.15 a. m. to 12.15 p. m., only

Write at top of first page of answer paper (a) name of school where you have studied, (b) number of weeks and recitations a week in (1) elementary algebra, (2) intermediate algebra, (3) advanced algebra. The minimum time requirement is five recitations a week in algebra for two school years.

Answer eight questions. Each answer should be reduced to its simplest form.

In the examination in advanced algebra the use of the slide rule will be allowed for checking, provided all computations with tables are shown on the answer paper.

1 The sum of the first two terms of a decreasing geometric series is $\frac{5}{4}$; the sum to infinity is $\frac{9}{4}$. Find the first three terms of the series. [12$\frac{1}{2}$]

2 Find the roots of the equation
$$3x^3 - 4x^2 + 2x + 4 = 0 \qquad [6\frac{1}{2}, 3, 3]$$

3 a Solve the equation $x^3 - 1 = 0$ and check one of the complex roots by substitution. [2, 4$\frac{1}{2}$]

b Represent the roots graphically and find their sum graphically. [3, 2]

c Check the result of the graphic addition by using one of the relations between the roots and the coefficients. [1]

4 a. Transform the equation $x^3 - 6x^2 + 4x + 8 = 0$ into another equation of the form $x^3 + px + q = 0$. [6$\frac{1}{2}$]

b Solve the resulting equation and from these roots obtain the roots of the given equation. [3, 1]

c Check by forming the equation having this last set of roots. [2]

5 a Find the value of $\left(\dfrac{0.004683}{0.06787}\right)^{\frac{3}{2}}$ [6$\frac{1}{2}$]

b If interest is compounded m times a year at the annual rate r, the amount A of P dollars after n years is given by the formula $A = P\left(1 + \dfrac{r}{m}\right)^{mn}$. Find the length of time it will take a sum of money to double itself if $r = 6\%$ and $m = 2$. [6]

6 A cork sphere 2 inches in diameter floating in water sinks to a depth x given by the equation $2x^3 - 6x^2 + 1.92 = 0$. Find the depth correct to the *nearest hundredth*. [12$\frac{1}{2}$]

7 a In how many ways can 8 men be arranged in a line if two of them, A and B, always stand together? [6]

b How many baseball nines can be chosen from 13 candidates, provided A, B, C and D are the only candidates for two positions and can play in no other positions? [6$\frac{1}{2}$]

8 Find the values of k that will make two values of x equal in the solution of the simultaneous equations $y = x^3$ and $y = 3x + k$. [12$\frac{1}{2}$]

9 a Prove that the equation $x^7 + x^4 - x^2 - 3 = 0$ has either 4 or 6 complex roots. [7$\frac{1}{2}$]

b Find the 7th term of the expansion $\left(a - \dfrac{1}{\sqrt{a}}\right)^{10}$ [5]

10 An open box is to be made out of a square piece of cardboard 6 inches on a side, by cutting out equal squares from the corners and then turning up the sides. If x represents a side of the square cut out from each corner, (a) express the volume y of the box in terms of x, (b) make a graph of this relation between y and x, (c) from the graph estimate the value of x that will give the largest box. [5, 5, 2$\frac{1}{2}$]

11 Solve the equations:
$$x^3 - y^3 = 19$$
$$x^2y - xy^2 = 6 \qquad [12\frac{1}{2}]$$

12 The circumferences of the wheels of a carriage are in the ratio 4 : 5. In going 1 mile the fore wheel makes 80 revolutions more than the hind wheel. Find the circumference of each wheel. [12$\frac{1}{2}$]