The University of the State of New York

231st High School Examination

ADVANCED ALGEBRA

Monday, June 16, 1924 — 9.15 a. m. to 12.15 p. m., only

Write at top of first page of answer paper (a) name of school where you have studied, (b) number of weeks and recitations a week in (1) elementary algebra, (2) intermediate algebra, (3) advanced algebra. The minimum time requirement is five recitations a week in algebra for two school years.

Answer eight questions. Each answer should be reduced to its simplest form.

In the examination in advanced algebra the use of the slide rule will be allowed for checking, provided all computations with tables are shown on the answer paper.

1 Solve the equation $x^4 - 3x^3 + 4x^2 + 3x + 7 = 0$, knowing that one of the roots is $2 + \sqrt{-3}$.   [12½]

2 a Find the value of $\dfrac{x}{x^2 - 17}$ when $x = 3 - 2\sqrt{-1}$. Express the result in the form $a + b\sqrt{-1}$ [9]

b Add graphically $2 + 3\sqrt{-1}$ and $-2 + \sqrt{-1}$ [3½]

3 a Derive the rule for transforming an equation of the nth degree into another equation in which the roots shall be those of the first equation multiplied by the constant m.   [6½]

b Transform the equation $24x^3 + 12x^2 + 8x + 6 = 0$ into another equation with integral coefficients, the coefficient of the highest degree term being unity.   [6]

4 By the use of Horner's method, find to the *nearest tenth* the positive root of $x^3 - 6x^2 + 9x - 17 = 0$ [12½]

5 a Find in how many years a sum of money (P) will treble itself at 5% if the interest is compounded annually, using the formula $A = P(1 + r)^n$ in which $A =$ the amount, $r =$ the interest on $1 for one year and $n =$ the number of years.   [6]

b Find the value of $\sqrt[6]{\left(\dfrac{0.06843}{0.8246}\right)^3}$ [6½]

6 The first two terms of a geometric progression are $\dfrac{b}{1+c}$ and $\dfrac{b}{(1+c)^2}$. Show that the sum of n terms of this progression is given by the formula $s = b\left[\dfrac{1 - (1+c)^{-n}}{c}\right]$ [12½]

7 a How many of the arrangements of the letters of the word *factoring* will begin with a vowel and end with a consonant?   [5½]

b How many words consisting of 2 vowels and 3 consonants can be formed from the letters of the word *factoring*, if any arrangement of letters is considered as a word?   [7]

8 Find the value of m in the equation $x^3 - 3x^2 + mx + 15 = 0$ if the roots are in arithmetic progression.   [12½]

9 Find the values of k that will make the two values of x equal in the solution of the simultaneous equations

$$x^2 + y^2 = 25$$
$$y = \tfrac{1}{3}x + k \qquad [12½]$$

10 In walking from town A to town C a man passes through town B midway between A and C. His rate of walking from A to B is ½ of a mile an hour more than his rate of walking from B to C and he arrives at C in 5½ hours. Returning from C to A he walks 1 mile an hour more than during his journey from A to B and it takes him 3¾ hours to make the return journey. Find the distance from A to C and the rate of walking from A to B.   [8½, 4]

11 a Plot between the values $x = -4$ and $x = +2$ the curve represented by the equation

$$y = 2x^3 + 3x^2 - 12x + 5 \qquad [9½]$$

b From the curve estimate the roots of the equation

$$2x^3 + 3x^2 - 12x + 5 = -2 \qquad [3]$$