The University of the State of New York

27TH HIGH SCHOOL EXAMINATION

ADVANCED ALGEBRA

Monday, June 19, 1922—9.15 a. m. to 12.15 p. m., only

Write at top of first page of answer paper (a) name of school where you have studied, (b) number of weeks and recitations a week in (1) elementary algebra, (2) intermediate algebra, (3) advanced algebra.

The minimum time requirement is five recitations a week in algebra for two school years.

Answer eight questions. Each answer should be reduced to its simplest form.

In the examination in advanced algebra the use of the slide rule will be allowed for checking, provided all computations with tables are given on the answer paper.

1 Solve the equation $x^4-6x^3+19x^2-26x+18=0$, knowing that one root is $2-\sqrt{-5}$ [12½]

2 *a* Find the value of $\frac{2x-3}{x^2-x+7}$ when $x=2-3i$ and express the result in the form $a+bi$. [4, 4]

b Add graphically $\frac{-1+\sqrt{-3}}{2}$ and $\frac{-1-\sqrt{-3}}{2}$ [2, 2, 2½]

3 *a* Derive the rule for transforming an equation into another whose roots are those of the first equation multiplied by the constant m. [5½]

b Form an equation with integral coefficients whose roots are the roots of $2x^4+10x^3-7x^2+x+4=0$ each multiplied by $\frac{1}{2}$. [5]

4 *a* Without solving, determine the nature of the roots of the equation $x^5+4x^4-3x^3-x+12=0$ [5½]

b Write the value of the sum of the roots, the product of the roots and the sum of the products taken three at a time. [2, 2, 2]

5 Locate all the roots of $2x^3+3x^2-9x-7=0$ and find the positive root to the *nearest* tenth. [3, 5½]

6 *a* How many different committees consisting of 3 seniors and 2 juniors can be selected from 8 seniors and 10 juniors? [5]

b If ${}_nP_r=20$ and ${}_nC_r=10$, find n and r. [5½]

7 Find the sum of all integers between 100 and 800 that are divisible by 3. [12½]

8 Find what values of k will make the two values of x equal in the solution of the simultaneous equations:

$$16x^2+25y^2=400$$
$$y=\frac{4}{5}x+k$$ [12½]

9 Two launches race over a course of 12 miles. The first travels at the rate of 7½ miles per hour. The other, which has a start of 10 minutes, runs over the first half of the course with a certain speed but increases its speed over the second half by 2 miles per hour, winning the race by a minute. Find the speed of the second launch over the first half of the course. [12½]

10 *a* Plot the graph of $y=x^3-9x^2+24x-7$ from $x=0$ to $x=6$ [5]

b From the graph determine a root of
$$x^3-9x^2+24x-7=0$$ [1½]

c From the graph determine the roots of
$$x^3-9x^2+24x-7=13$$ [3]

d From the graph determine the nature of the roots of
$$x^3-9x^2+24x-7=25$$ [3]

11 *a* Write the first *four* terms in the expansion of
$$\left(\frac{x^2}{2}-2y\right)^{16}$$ [5½]

b Without expanding, find the eighth term and simplify the result. [5]