ADVANCED ALGEBRA

Monday, June 13, 1921—9.15 a. m. to 12.15 p. m., only

Write at top of first page of answer paper (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in (1) elementary algebra, (2) intermediate algebra, (3) advanced algebra. The minimum time requirement is five recitations a week in algebra for two school years.

Answer eight questions. Each answer should be reduced to its simplest form.

1 If one root of the equation $x^3 + x^3 - 12x^2 + 32x - 40 = 0$ is $1 - \sqrt{-3}$ find the other roots. [12½]

2 *a* By use of a formula express .13636 . . . as a common fraction. [5½]

  b Deduce the formula for the sum of n terms of an arithmetical progression in terms of the first term, the last term and the number of terms. [7]

3 Solve for x and y and correctly group your answers:
$$x^2 - xy = 27y$$
$$xy - y^2 = 3x$$
  Solution [11], grouping [1½]

4 Solve the following equation for the values of x:
$$\sqrt{mn - x} - \sqrt{x}\sqrt{mn - 1} = \sqrt{mn}\sqrt{1 - x}$$ [12½]

5 Transform the equation $x^3 + 6x^2 + 3x + 8 = 0$ into an equation lacking the second degree term. Express the relation between the roots of this equation and the original equation. [9], [3½]

6 *a* Rationalize the denominator of $\frac{\sqrt{5} + \sqrt{3}i}{\sqrt{5} - \sqrt{3}i}$ [4½]

  b Find the cube roots of 8 and represent them graphically. [4, 4]

7 Obtain all the information possible concerning the roots of each of *three* of the following equations by the use of Descartes' rules and by inspection of the constant terms:

  a $2x^3 + 3x^2 + 7 = 0$

  b $3x^4 + 7x^2 + 4 = 0$

  c $3x^3 - x^2 + 2x - 1 = 0$

  d $x^6 - 6x^4 - 8x^2 + 9 = 0$ [12½]

8 If the dining tables in a boys' boarding school of 60 students seat six persons each, how many different groups of students can sit at the table with the head master? [12½]

9 To what is the log $\sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$ equal?

By use of logarithms find the value of this expression when $a = 54.7$, $b = 65.3$, $c = 83.4$ and $s = \frac{a+b+c}{2}$ [3½], [9]

10 A man rows down stream 12 miles in 4 hours less time than it takes him to return; should he row at twice his ordinary speed in still water his rate down stream would be 10 miles per hour. Find (*a*) his usual rate of rowing in still water, (*b*) the rate of the stream. (*a*) [9½], (*b*) [3]

11 By the use of Horner's method, find to the *nearest* hundredth the root lying between 5 and 6 of the equation
$$x^3 + x^2 - 25x - 81 = 0$$
  [12½]; [10] if not written to the nearest hundredth

12 Solve graphically the following set of equations:
$$x^2 + y^2 = 9$$
$$y = x^2 - 5x + 6$$
  Representing first equation [2], second equation [7½], reading solution [3]