The University of the State of New York

221st High School Examination

ADVANCED ALGEBRA

Monday, June 16, 1919—9.15 a. m. to 12.15 p. m., only

Answer eight questions. Each answer should be reduced to its simplest form.

1 Represent graphically

  a $2 + 3\sqrt{-1}$

  b   The sum of $3 + 4\sqrt{-1}$ and $-2 - 3\sqrt{-1}$

  c   The sum of $-\frac{1}{2} + \frac{1}{2}\sqrt{-3}$, 1, and $-\frac{1}{2} - \frac{1}{2}\sqrt{-3}$

2 Solve *one* of the following equations:

$$\frac{a^2}{x^2} + \frac{2a}{x} = 3$$

$$ax - \frac{b}{x} = a - b$$

3 Solve the following simultaneous equations:

$$\begin{cases} x^2 + xy + 2y^2 = 11 \\ 2x^2 - xy + 2y^2 = 8 \end{cases}$$

4 *a* How many different permutations can be formed from the letters x, x, x, y, z and w, taken altogether?

  b How many diagonals has a convex polygon of n sides? Check by drawing figures to illustrate the cases of $n = 3$ and $n = 4$

5 By the use of graphs find to the nearest tenth each root of the following simultaneous equations:

$$\begin{cases} y^2 - 2x = 4 \\ 2x^2 + y = 3 \end{cases}$$

6 Find to *three* decimal places the root of the equation $x^3 + 5x^2 - 7x + 2 = 0$ that lies between 0.4 and 0.5.

7 Prove that an equation of the nth degree can have no more than n roots.

8 Solve the equation $x^3 + 3x^2 - 25x - 12 = 0$ for all roots. Check the roots.

9 All the roots of the equation $x^3 - 12x^2 + 47x - 60 = 0$ are real; determine their signs.

10 Prove that the area of a square figure can not be increased by increasing one dimension and decreasing the other by the same amount.

11 It is proved in geometry that $\overline{PT}^2 = PA \times PB$ where PT is the tangent from the external point P and where PB is any line from P meeting the circle in A and B; if it is known that $PA = 2$ and $PB = 1 + PT$, what is the length of PT?

12 If the edges of a rectangular box are increased by 2 inches, 3 inches and 4 inches respectively, the box becomes a cube and the capacity is increased by 1008 cubic inches; find the dimensions of the box before the increase.

13 The time of the fall of a bomb from an airplane is given by the formula $T = \frac{\sqrt{H}}{4} + \frac{H}{9000}$ where T is the number of seconds and H is the height in feet; find the value of H in terms of T and thus obtain a formula for the height of the airplane when the time of descent of the bomb is known.

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DIRECTIONS FOR RATING

The direction, "Less than 60% of the credit should be granted when an error in computation occurs," should be followed in rating all incorrect answers to questions which fall under the topics mentioned in "Suggestions on the Rating of Regents Examination Papers in Mathematics" under "General 3."

Except in schools where the "committee system" is used, teachers are urged to mark papers cumulatively, that is, to add the credits earned by each answer to the total credits earned by preceding answers so that the mark given to the last answer is the per cent to which the paper is entitled, e. g. consecutive answers earning 5, 7, 4 etc. respectively should be marked 5, 12, 16 etc. respectively.

1 $12\frac{1}{2}$ credits
 a Allow 3 credits.
 b Allow 4 credits.
 c Allow $5\frac{1}{2}$ credits.

2 $12\frac{1}{2}$ credits
 Allow $7\frac{1}{2}$ credits for first correct result.
 Allow 5 credits for other correct result.

3 $12\frac{1}{2}$ credits
 Allow $7\frac{1}{2}$ credits for first two pairs of roots.
 Allow 5 credits for the other two pairs of roots ($2\frac{1}{2}$ each).

4 $12\frac{1}{2}$ credits
 a Allow $6\frac{1}{2}$ credits.
 b 6 credits. Allow 4 credits for finding the number of diagonals. Allow 2 credits for drawing figures (1 each).

5 $12\frac{1}{2}$ credits
 See Suggestion 19.
 Allow $6\frac{1}{2}$ credits for correct graphs.
 Allow 6 credits for determining the roots to the nearest tenth ($1\frac{1}{2}$ for each pair).

6 $12\frac{1}{2}$ credits
 Allow 1 credit for first figure of root.
 Allow 5 credits for second figure of root.
 Allow $6\frac{1}{2}$ credits for third figure of root.

7 $12\frac{1}{2}$ credits
 See General Suggestions 2 and 7.

8 $12\frac{1}{2}$ credits
 Allow $6\frac{1}{2}$ credits for finding the roots.
 Allow 6 credits for checking.

9 $12\frac{1}{2}$ credits
 Allow $6\frac{1}{2}$ credits for finding the number of positive roots.
 Allow 6 credits for finding the number of negative roots.

10 $12\frac{1}{2}$ credits
 See General Suggestion 7.

11 $12\frac{1}{2}$ credits
 See General Suggestion 3.

12 $12\frac{1}{2}$ credits
 Allow $6\frac{1}{2}$ credits for correct equation.
 Allow 6 credits for finding the dimensions.

13 $12\frac{1}{2}$ credits
 See General Suggestion 3.