The University of the State of New York

213TH HIGH SCHOOL EXAMINATION

ADVANCED ALGEBRA

Monday, June 14, 1915—9.15 a. m. to 12.15 p. m., only

Write at top of first page of answer paper (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in (1) elementary algebra, (2) intermediate algebra, (3) advanced algebra.

The minimum time requirement is five recitations a week in algebra for two school years.

Answer eight questions. Each answer should be reduced to its simplest form.

1 *a* From a fleet of 6 dreadnoughts and 10 cruisers a squadron of 2 dreadnoughts and 4 cruisers is to be made up; in how many ways can this be done?

b Through the telephone system of a small village 435 conversations (not all at the same time) are possible between different instruments; how many instruments are there in the village?

2 *a* Put *each* of the following expressions in one of the forms $+p$, $-p$, $+ip$, or $-ip$, when p is a positive real number:

$\sqrt{-18} \times \sqrt{-50}$

$\sqrt{-15} : \sqrt{-5}$

$1\sqrt{-25}$

$(3\sqrt{-5})^3$

$(-1) : i^{19}$

b Put *each* of the following expressions in the form $a+ib$, when a and b are real numbers: $(2-3i)(3+4i)$; $(1+i)^2$; $100\ (3+4i)$

3 On a holiday trip there were two more women than men; in paying the expenses each man gave 50 cents more than each woman, the men together paying $27 and the women $20. How many men and how many women went on the trip?

4 Reduce $\frac{12-5i}{2-3i}$ and $\frac{-5}{1+2i}$ to simplest form and represent graphically their sum and their difference.

5 For what values of k will the quadratic

$$2x^2-3(x+k)^2+4x-6(x+k)-25=0$$

have equal roots? Find these roots.

6 $3x^5 - 17x^4 - 17x^3 - 17x^2 - 11x - 60 = 0$

a Give a reason for *each* of the following statements about the above equation:

(1) The equation has at least one positive real root.

(2) The equation has not more than one positive real root.

b Find the positive real root, assuming that it lies between $3\frac{1}{2}$ and 11 and is rational.

7 Two of the roots of an equation of the n^{th} degree are i and $\sqrt{5}$. If the coefficients are real but not rational, what is the least value possible for n? If the coefficients are real and rational, what is the least value possible for n? Form the equation in each case.

8 *a* Solve $x^2 - 2x - 4\sqrt{x^2 - 2x + 6} + 9 = 0$ and check the complex roots.

b What is the condition that gives $ax^2 + bx + c$ the same sign for all values of x?

9 Plot for integral values of x between $x = -2$ and $x = +5$ the equation $y = x^3 - 3x^2 - 4x - 10$; compute, by Horner's method, to *two* places of decimals, the positive root of the right hand member set equal to zero.

10 Determine r so that one root of $x^3 - 6x^2 - 13x + r = 0$ may be equal to half the sum of the other two roots. Solve the equation.