

ADVANCED ALGEBRA

Monday, June 17, 1912—9.15 a. m. to 12.15 p. m., only

Write at top of first page of answer paper (a) name of school where you have studied, (b) number of weeks and recitations a week in algebra.

The minimum time requirement is five recitations a week in algebra for two school years.

Answer eight questions. Credit will not be granted unless all operations (except mental ones) necessary to find results are given; simply indicating the operations is not sufficient. Each answer should be reduced to its simplest form.

1 How many different sums of money can be made up from a cent, a nickel, a dime and a quarter? [Solve by permutations or combinations.]

2 What is the distinction between a permutation and a combination?

Deduce the formula for the number of combinations of n things taken r at a time.

3 Show that $(2+i)^2 = \frac{7+i}{1-i}$ when $i = \sqrt{-1}$

4 How many terms are there in the development of a determinant of the n th order? How are these terms formed? How is the sign of any term determined?

5 Find by the use of determinants the value of y in the following system of equations:

$$\begin{aligned}x + 3y - z &= 1 \\y + 3z - w &= 4 \\z + 3w - x &= 11 \\w + 3x - y &= 2\end{aligned}$$

6 Without solving the equation, state *five* facts in regard to the roots of the equation $x^5 + 2x^3 + x^2 - x - 1 = 0$

7 Find *all* the roots of the equation

$$x^4 - 6x^3 + 6x^2 + 10x - 3 = 0$$

8 Plot for integral values of x between $x=0$ and $x=5$ the equation $y = x^3 - 2x^2 - 4x - 5$ and determine, to *two* places of decimals, the positive root of the right hand member set equal to zero.

9 An equation with rational coefficients is known to have as roots the numbers $\sqrt{2}$, 3 , $1-i$; what other roots must this equation have? Assuming that this equation is of the lowest possible degree, find the coefficients of the x term in next to the highest power of x and find the constant term.

10 If the roots of the equation $ax^2 + bx + c = 0$ are in the ratio of $m:n$ show that $mnb^2 = (m+n)^2ac$