189TH HIGH SCHOOL EXAMINATION

ADVANCED ALGEBRA

Monday, June 11, 1906—9.15 a. m. to 12.15 p. m., only

Answer eight questions. Give each step of solution. Reduce fractions to lowest terms. Express final result in its simplest form and mark it Ans. Each complete answer will receive 12½ credits. Papers entitled to 75 or more credits will be accepted if written by students in class A; those entitled to 60 or more credits will be accepted if written by students in class B.

1 Simplify $\frac{3\sqrt{-5}+2\sqrt{-2}}{3\sqrt{-5}-2\sqrt{-2}}$; $\sqrt{5-12\sqrt{-1}}$

2 Expand $\left(a-3x^{-\frac{3}{2}}\right)^{-\frac{1}{3}}$ to *five* terms by the binomial formula.

3 Prove that the number of combinations of n things taken r at a time is equal to the number of combinations of n things taken $n-r$ at a time. Illustrate this principle in the solution of the following: Out of 21 men in how many ways can 18 be chosen?

4 Divide $a^5-3a^2b^3+8ab^4-5b^5$ by $a^2-4ab+b^2$ by synthetic division, finding the quotient to *four* terms and the remainder.

5 In the series $1+\frac{3^2}{2}+\frac{3^3}{3}+\frac{3^4}{4}+\ldots$ find the limiting ratio between the $(n+1)$th term and the nth term. Determine whether the series is convergent or divergent.

6 Represent graphically $a\sqrt{-1}$ and $a+b\sqrt{-1}$, giving a complete explanation of each diagram.

7 Expand $\sqrt{1-3x+x^2}$ to *four* terms by the method of undetermined coefficients.

8 Two trains A and B 360 miles apart start at the same time and travel toward each other; train A reaches B's starting point 4 hours after passing B, train B reaches A's starting point 9 hours after passing A. Find the velocity of each train.

9 Prove that $\log_b m=\frac{\log_a m}{\log_a b}$

10 Find by the method of differences the sum of the first n terms of the series 1^3, 2^3, 3^3, 4^3, 5^3 . . .

11 State Descartes's rule of signs and apply it in proving that $x^7-3x^4+2x^3-7=0$ must have at least four imaginary roots.

12 Evaluate the determinant $\begin{vmatrix} 2 & 6 & -1 \\ 4 & -2 & -5 \\ 3 & 0 & 7 \end{vmatrix}$ <-3 0 7