January 25, 1967

Part I

Answer all questions in this part. Each correct answer will receive 2½ credits. No partial credit will be allowed. Write your answers in the spaces provided.

1. If the probability that a basketball player will succeed in making a certain shot is 0.4, what is the probability that he will *not* succeed? 1_____

2. Solve for x: $4^{\frac{3x}{2}} = 2^{x+4}$ 2_____

3. Express $\dfrac{1}{2 + i\sqrt{3}}$ as an equivalent fraction with a real denominator. 3_____

4. Solve for y: $\sqrt{3y - 8} = y - 6$ 4_____

5. If $(a + bi) + (2 - i) = 3 + i$, find the value of b. 5_____

6. The height(s) above the ground, in feet, of a ball thrown vertically upward is given by the equation $s = 44 + 80t - 16t^2$, where t is measured in seconds. Find the velocity of the ball in feet per second when $t = 1$. 6_____

7. How many distinct five-letter arrangements can be made from the letters of the word "ADDED"? 7_____

8. Find the rational root of the equation $2x^3 - x^2 - x - 3 = 0$. 8_____

9. A root of $x^3 + x^2 + x - 7 = 0$ lies between 1.4 and 1.5. Find this root to the *nearest tenth*. 9_____

10. Find the values of x which satisfy the inequality $3 + x \leq 4x - 5$. 10_____

11. Find the remainder, independent of x, when $ax^2 + bx + c$ is divided by $x - r$. 11_____

12. For what value of k is 2 a root of the equation $2x^4 - 6x^3 + 4kx + 13 = 0$? 12_____

13. F varies directly as m and inversely as d^2. If $F = 96$ when $m = 4$ and $d = 6$, find F when $m = 6$ and $d = 8$. 13_____

14. How many committees each consisting of 3 boys and 2 girls can be chosen from a group of 8 boys and 5 girls? 14_____

15. Find the numerical value of $\log_{16} 64$. 15_____

16. If the graphs of the equations $4y - 2x = 7$ and $ax + 5y = 10$ are perpendicular to each other, find the numerical value of a. 16_____

17. Solve for x in terms of y: $y = \dfrac{2x + 7}{x - 3}$ 17_____

18. The cost of a telephone call is a cents for the first three minutes and b cents per minute for each minute thereafter. If n is an integer greater than 3, write an expression for the cost of a call for n minutes. 18_____

19. The 5th term of an arithmetic progression is s and the 15th term is t. Find the common difference in terms of s and t. 19_____

Directions (20-24): Indicate the correct completion for *each* of the following by writing the number 1, 2, 3, or 4 in the space provided.

20. One of the equations for the family of lines passing through the point whose coordinates are (0, —3) is (1) $y = -3x + b$ (2) $y = b$ (3) $y = mx - 3$ (4) $y = mx$ 20_____

21. The sum of the roots of the equation $x^3 - 3x^2 + 2x - 1 = 0$ exceeds the product of the roots by (1) 1 (2) 2 (3) —2 (4) 4 21_____

22. The graph of the equation $3x^2 + 12x - 20y + 42 = 0$ is (1) a parabola (2) an ellipse (3) a circle (4) a hyperbola 22_____

23. The coordinates of the point of inflection of $y = \frac{x^3}{6} + \frac{x^2}{2} + 2x - 1$ are (1) $\left(-1, -\frac{3}{2}\right)$ (2) $\left(-1, -\frac{8}{3}\right)$ (3) $\left(1, \frac{5}{3}\right)$ (4) $\left(1, \frac{7}{2}\right)$ 23_____

24. The numerical value of $10^{\log 3}$ is (1) $\frac{10}{3}$ (2) $\frac{3}{10}$ (3) 3 (4) $\frac{1}{3}$ 24_____

Part II

*Answer sixteen questions from this part, 25-48. Each correct answer will receive 2½ credits. No partial credit will be allowed. Questions marked * are based upon optional topics in the syllabus. Write your answers in the space provided.*

25. Given i is the imaginary unit, write $(-i)^{56}$ in *simplest form.* 25_____

26. Determine all the values of x for which the inequality $x^2 - x - 6 < 0$ is true. 26_____

*27. The area of a triangle, expressed in the form $\frac{1}{2}\begin{vmatrix} 2 & 0 & 1 \\ 3 & 5 & 1 \\ x & 8 & 1 \end{vmatrix}$ is 14. Find the value of x. *27_____

28. Four points A, B, C, and D, which represent complex numbers plotted in the complex plane, are the vertices of parallelogram $ABCD$. If A represents $0 + 0i$, B represents $5 + i$, and D represents $2 + 7i$, what complex number does C represent? 28_____

29. Multiply $2(\cos 115° + i \sin 115°)$ by $6(\cos 245° + i \sin 245°)$ and express the result in the rectangular form, $a + bi$. 29_____

30. If $y = x^2 + 3x - 2$, find the average rate of change of y with respect to x as x increases from $x = 1$ to $x = 4$. 30_____

31. What is the abscissa of the point on the graph of $y = x^2 + 5x + 4$ where the slope of the tangent equals 9? 31_____

32. The arithmetic mean between two numbers is -6 and their positive geometric mean is $4\sqrt{2}$. Find the two numbers. 32_____

33. What is the sum of the seven numerical coefficients in the expansion of $(a + b)^6$? 33_____

34. Express the repeating decimal 0.4333..., in which the digit 3 is repeated endlessly as indicated, in the form $\frac{a}{b}$ where a and b are integers. 34_____

35. If $f(x) = x^2 + 2x - 3$, write $f(a - 3)$ as an expression free of parentheses. 35_____

*36. Change $x^2 + y^2 = 2x + 15$ from rectangular coordinates to polar coordinates. *36_____

37. If $\log_{10} e = 0.4343$, find $\log_e 100$ to the *nearest tenth*. 37_____

38. What is the abscissa of the point at which the graph of $y = -2x^2 + x + 3$ is intersected by its axis of symmetry? 38_____

39. Write an equation of the line whose x-intercept and y-intercept are each twice the corresponding intercepts of the graph of the equation $5x - 2y = 10$. 39_____

40. The endpoints of a diameter of a circle are (6, 0) and (0, 8). Write the equation of this circle in the form $(x - h)^2 + (y - k)^2 = r^2$. 40_____

41. The equation $x^3 + 6x^2 + 13x + 10 = 0$ has a root $x = -2$. Express *one* of the remaining roots in the form $a + bi$. 41_____

Directions (42-48): For each *of those chosen*, write in the space provided the *number* preceding the expression that best completes *each* statement or answers *each* question.

42. A rectangle is twice as long as it is wide. Its diagonal is d inches long and its area contains A square inches. The area A expressed in terms of d is (1) $\frac{2d^2}{3}$ (2) $\frac{3d^2}{2}$ (3) $\frac{2d^2}{5}$ (4) $\frac{5d^2}{2}$ 42____

43. The value or values of x for which the expression $\frac{x - 1}{2x(x + 1)}$ is undefined would be (1) 1 only (2) -1 only (3) 0 only (4) 0 and -1 43____

44. Which point does *not* lie on the graph of $y = \log_4 x$?
(1) $(\frac{1}{2}, -\frac{1}{2})$ (2) (1, 0) (3) (−4, −1) (4) $(2, \frac{1}{2})$ 44____

45. In how many points do the graphs of $xy = 6$ and $(x + 3)^2 + (y - 3)^2 = 4$ intersect? (1) 1 (2) 2 (3) 0 (4) 4 45____

46. The roots of the equation $x^2 - kx + k = 0$ are real and unequal if (1) $k < 0$ or $k > 4$ (2) $0 < k < 4$ (3) $k = 0$ (4) $k = 4$ 46____

47. It requires 8 hours for machine A to do a certain job alone, and it requires 15 hours for machine B to do the same job alone. If two machines of exactly the same type as A and three machines of exactly the same type as B work on this job together, in how many hours will they complete it? If x represents the time required for these 5 machines to do the job together, then a correct equation for the solution of this problem is

(1) $\frac{x}{16} + \frac{x}{45} = 1$ (2) $\frac{1}{4} + \frac{1}{5} = \frac{5}{x}$

(3) $\frac{x}{8} + \frac{x}{15} = 1$ (4) $\frac{x}{4} + \frac{x}{5} = 1$ 47____

48. A complex root of $x^5 + 32 = 0$ is (1) $2(\cos 72^\circ + i \sin 72^\circ)$ (2) $2(\cos 36^\circ + i \sin 36^\circ)$ (3) $-2(\cos 108^\circ + i \sin 108^\circ)$ (4) $-2(\cos 36^\circ + i \sin 36^\circ)$ 48____