

TWELFTH YEAR MATHEMATICS

12A (Advanced Algebra)

Thursday, January 24, 1963—1:15 to 4:15 p.m., only

Part I

Answer all questions in this part. Each correct answer will receive $2\frac{1}{2}$ credits. No partial credit will be allowed. Write your answers in the spaces provided on the separate answer sheet.

- Express $\frac{3 + 2i\sqrt{3}}{3 - i\sqrt{3}}$ as an equivalent fraction with a real denominator.
- Find the length of the radius of the circle defined by $x^2 + y^2 + x - 6y - 12 = 0$.
- Write an equation of the line through the point (1, 2) which is perpendicular to the line through the points (-4, 1) and (6, -3).
- If the turning point of the parabola defined by $y = kx^2 + 4x - 4$ is a point on the x -axis, find the value of k .
- If $y = x^2 + 4x - 8$, find the average rate of change of y with respect to x as x increases from $x = 1$ to $x = 4$.
- If $x^2 = \frac{1 - y}{1 + y}$, express y as a function of x .
- Express the complex number $8(\cos 330^\circ + i \sin 330^\circ)$ in the form $a + bi$.
- A mixture of water and acid which is one-fifth acid contains p liters. To this mixture s liters of water are added. Express in terms of p and s the fractional part of the new mixture which is acid.
- Express in polar form the product of $2(\cos 53^\circ + i \sin 53^\circ)$ and $(\cos 82^\circ + i \sin 82^\circ)$.
- Express in polar form the root of $x^3 - 64 = 0$ which, if plotted, would lie in quadrant II.
- Write an equation of the line which is tangent to the curve defined by $y = 3x^2 + 2$ at the point (1, 5).
- Find the values of x for which $\log(x + 1) + \log(x - 1) = \log(2x^2 - 5x + 5)$.
- The probability of getting exactly 6 heads in 10 tosses of a coin may be found by evaluating the *fifth* term of the expansion of $(\frac{1}{2} + \frac{1}{2})^{10}$. Evaluate this term.
- What is the sum of the roots of the equation $2x^3 + 10x^2 + 1 = 0$?
- If $f(x) = 3x^2$, express the value of $\frac{f(2+h) - f(2)}{h}$ as a polynomial in h .
- If $(x - 2)$ is a factor of $(x - 2)^3 + 2(x - 2)^2 + k - 4$, find the value of k .
- The electrical resistance of a cable varies directly as its length and inversely as the square of its diameter. If the resistance of 5,000 feet of

cable, 0.5 inch in diameter, is 0.25 ohm, find to the nearest thousandth the number of ohms in the resistance of a cable of the same material which is 2,700 feet long and 0.3 inch in diameter.

18. A committee of 3 persons is to be chosen by lot from a group of 6 men and 6 women. What is the probability that the committee will consist of 2 men and 1 woman?

19. Find the rational root of $3x^3 + 2x^2 + 3x + 2 = 0$.

20. Find the real value of k such that $1 - i$ is a root of the equation $x^3 - 6x^2 + kx - 8 = 0$.

Directions (21-24): Indicate the correct completion for each of the following by writing the number 1, 2, 3 or 4 in the space provided on the separate answer sheet.

21. If $4^x = \sqrt{2^{3y}}$, then (1) $x = 3/4y$ (2) $x = 3y$ (3) $y = 3/4x$
(4) $x = 1/3y$

22. The entire set of values of x which satisfies the inequality $5 - 3x < 2$ also satisfies the inequality (1) $x > \frac{7}{3}$ (2) $x < -\frac{7}{3}$ (3) $x > 1$
(4) $x < 1$

23. When drawn on the same axes, the graphs of $x^2 + 4y^2 = 16$ and $y = x^2 + 2$ have in common exactly (1) one point (2) no points
(3) three points (4) four points

24. The point (1, 1) on the graph of $y = 1/3x^3 - x^2 + x + 2/3$ is (1) a relative maximum point (2) a relative minimum point (3) a point of inflection (4) neither a maximum or minimum point nor a point of inflection

Part II

*Answer sixteen questions from this part, 25-48. Each correct answer will receive $2\frac{1}{2}$ credits. No partial credit will be allowed. Questions marked * are based upon optional topics in the syllabus. Write your answers on the separate answer sheet.*

25. If $f(x) = x + 3$ and $g(y) = y^2$, find $g[f(3)]$.

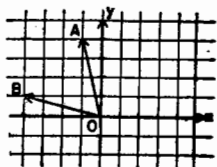
26. Find the coordinates of the point of intersection in quadrant IV of the graphs of $x^2 = 2y + 10$ and $3x - y = 9$.

27. Express $(2 - 3i)^2$ in the form $a + bi$.

*28. Express $(-3, -3\sqrt{3})$ in polar coordinates.

*29. If $\begin{vmatrix} 1 & a & -1 \\ 2 & -1 & 3 \\ 3 & -2 & 3 \end{vmatrix} = 10$, find the value of a .

30. On the accompanying graph, vectors OA and OB locate the points which represent the complex numbers $-1 + 4i$ and $-4 + i$, respectively. On the graph on the answer sheet, draw the vector(s) so as to indicate a graphical process for locating point C which will represent $(-1 + 4i) - (-4 + i)$. (Label point C .)



31. If \$100 is invested for a period of 10 years at 4% compounded semiannually, find the amount to the nearest dollar.

*32. Transform $r^2 - 4r \cos \theta = 0$, where r and θ denote polar coordinates, into an equation in rectangular coordinates.

33. In how many orders can 5 students be arranged in a straight line for a group picture?

34. A root of $x^3 + x - 4 = 0$ lies between 1 and 2. Find this root to the nearest integer.

35. Express $0.3666\dots$, where 6 is repeated endlessly, as a number in the form $\frac{a}{b}$ where a and b are integers.

36. Find the 8th term of the geometric progression $2\sqrt{3}, 6, 6\sqrt{3}, \dots$

37. An object rolling down an incline moves according to the equation $s = 20t + 8t^2$, where t represents the time in seconds and s represents the distance in feet of the object from the starting point at the top of the incline. Find the velocity of the object in feet per second when $t = 2$.

38. The perimeter of a square is 12 inches longer than that of a second square, and its area exceeds that of the second square by 39 square inches. Find the number of inches in the length of one side of the smaller square.

39. The equation of the axis of symmetry of the graph of $y = ax^2 - 6x + 5$ is $x = 1$. Find the value of a .

40. Write an equation which defines the family of lines passing through the point $(4, 0)$, excluding the line parallel to the y -axis.

41. The complex number $2i$ may be represented in the form $2(\cos 90^\circ + i \sin 90^\circ)$. Use this fact to find in polar form a value of $\sqrt{2i}$.

42. Express $\frac{\sqrt{2}}{1 + 1}$ as a number in the form $a + b\sqrt{2}$, where a and b are integers.

43. Find the number of diagonals in a regular polygon of 100 sides.

44. Solve the inequality: $x^2 - 3x - 4 < 0$

Directions (45-48): For each of those chosen, write in the space provided on the separate answer sheet, the number preceding the expression that best completes the statement.

45. The equation $\sqrt{y+5} = 7 - y$ has (1) only one real root (2) two positive roots (3) one positive root and one negative root (4) no real roots

46. All lines of the family defined by $y = kx + 3$, where k , x and y are real, (1) have the same slope (2) pass through the point $(1, 3)$ (3) have the same y -intercept (4) have the same x -intercept

47. The graphs of the functions defined by $y = 2^x$ and $y = 2^{-x}$ are (1) symmetric to each other with respect to the x -axis (2) symmetric to each other with respect to the y -axis (3) symmetric to each other with respect to the origin (4) not symmetric to each other

48. The ratio $\frac{\log_{10} x}{\log_5 x}$ is equivalent to (1) $\log_{10} 5$ (2) $\log_5 10$ (3) $\log_{10} x$ (4) $\log_5 x$

ANSWER SHEET

All of your answers should be recorded on this answer sheet.

Part I

Answer all questions in this part.

- | | | |
|---------|----------|----------|
| 1. | 9. | 17. |
| 2. | 10. | 18. |
| 3. | 11. | 19. |
| 4. | 12. | 20. |
| 5. | 13. | 21. |
| 6. | 14. | 22. |
| 7. | 15. | 23. |
| 8. | 16. | 24. |

Your answers for part II should be placed in the proper spaces on this sheet.

Part II

Answer only sixteen questions from this part. Be sure to write in the properly numbered spaces the answers to the questions you have chosen. Leave blank the spaces for the questions you do not choose to answer.

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|----------|----------|----------|
| 25. | 32. | 41. |
| 26. | 33. | 42. |
| 27. | 34. | 43. |
| 28. | 35. | 44. |
| 29. | 36. | 45. |
| 30. | 37. | 46. |
| 31. | 38. | 47. |
| | 39. | 48. |
| | 40. | |

