

January 25, 1961

## Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answer on the line at the right.

1. Find the remainder if  $3x^8 + 6x^3 + 7$  is divided by  $x + 1$ . 1\_\_\_\_\_
  2. Express the discriminant of the equation  $x^2 + mx - m = 0$  as a function of  $m$ . 2\_\_\_\_\_
  3. Solve for  $x$ :  $4^{2x-1} = \sqrt[3]{2}$  3\_\_\_\_\_
  4. Write an equation of the circle whose center is at  $(-3, 4)$  and whose radius is 5. 4\_\_\_\_\_
  5. Find the value of  $\sqrt[3]{1.34}$  to the nearest hundredth. 5\_\_\_\_\_
  6. Express  $0.23333\dots$  as a common fraction. 6\_\_\_\_\_
- Directions (7-10): These questions refer to  $\triangle ABC$  where  $A$ ,  $B$  and  $C$  are the points  $(-1, 2)$ ,  $(5, 4)$  and  $(2, 7)$ , respectively.
7. Find the slope of the altitude from  $C$  to  $AB$ . 7\_\_\_\_\_
  8. Find the length of  $AC$ . 8\_\_\_\_\_
  9. An equation of the line through  $C$  and  $B$  is  $x + y = 9$ . Write an equation of the family of lines parallel to  $CB$ . 9\_\_\_\_\_
  10. Find the coordinates of the point where the line through  $C$  and  $B$  intersects the  $x$ -axis. 10\_\_\_\_\_
  11. If the product of the roots of the equation  $ax^2 + 4x - 9 = 0$  is 3, find the sum of the roots. 11\_\_\_\_\_
  12. If  $f(x) = x^{-2/3} + 8x^0 - x^{-1}$ , find  $f(64)$ . 12\_\_\_\_\_
  13. Find the maximum value of  $y$  if  $y = -2x^2 + 12x - 18$ . 13\_\_\_\_\_
  14. Find the abscissa of the point where the graph of  $y = \log_3 x$  intersects the  $x$ -axis. 14\_\_\_\_\_
  15. Between what two consecutive integers does the positive root of  $x^4 + 2x^2 - 6 = 0$  lie? 15\_\_\_\_\_
  16. Find in simplest form the fourth term of the geometric progression  $\sqrt[3]{2}$ ,  $\sqrt{2}$ ,  $\sqrt[3]{8}$ ,  $\dots$  16\_\_\_\_\_
  17. Find the value of  $\log_{16} \frac{1}{8}$ . 17\_\_\_\_\_
  18. If  $y = 2x^2 - x + 1$ , find the average rate of change of  $y$  with respect to  $x$  in the interval from  $x = 2$  to  $x = 5$ . 18\_\_\_\_\_
  19. Write in simplest form the ninth term in the expansion of  $(a^{1/2} + x^{1/2})^{10}$ . 19\_\_\_\_\_
  20. It took 12 men 5 hours to build an airstrip. If 4 more men had been hired to work at the same rate, how many hours would the job have taken? 20\_\_\_\_\_

21.  $A$  is a member of a group of 10 from which a committee of 3 is to be chosen at random. What is the probability that  $A$  will be a member of this committee? 21\_\_\_\_\_

22. How many odd numbers of three digits each can be formed from the digits 2, 4, 6 and 7, if repetitions are permitted? 22\_\_\_\_\_

*Directions (23-25): Write on the line at the right of each of the following the number preceding the expression that best completes the statement or answers the question.*

23. Which of the following is a rational integral function of  $x$ ?

(1)  $2x^6 + 3$     (2)  $3x^{1/2} - 1$     (3)  $4 - \frac{1}{x^2}$     (4)  $17\sqrt[3]{x} + 1$     23 2

24. If  $x$  is divided by  $y$ , the quotient is 3 and the remainder is 2.

This relation may be expressed by (1)  $\frac{x}{y} = 3\frac{2}{y}$

(2)  $\frac{x}{y} = 3y + 2$     (3)  $\frac{x}{y} = 3y + \frac{2}{y}$     (4)  $\frac{x}{y} = 3 + \frac{2}{y}$     24\_\_\_\_\_

25. If the first term of an arithmetic progression is  $a$  and the common difference is  $2a$ , the sum of the first  $n$  terms is

(1)  $n^2$     (2)  $2an^2$     (3)  $an^2$     (4)  $a^2n^2$     25\_\_\_\_\_

## Part II

*Answer ten questions from this part. Each correct answer will receive  $2\frac{1}{2}$  credits. No partial credit will be allowed. Questions marked \* are based upon optional topics in the syllabus. Write your answer on the line at the right.*

26. What is the rational root of the equation  $5x^3 - x^2 + 5x - 1 = 0$ ? (-1/5) 26\_\_\_\_\_

27. Given: A parabola whose equation is  $y = 2x^2 - 3x - 2$ . Find the slope of the line tangent to the curve at the point  $(2, 0)$ . 27\_\_\_\_\_

28. The roots of the equation  $x^3 + px^2 + qx + r = 0$  are  $2$ ,  $-i$  and  $i$ . Find the value of  $q$ . 28\_\_\_\_\_

29. If the polynomial  $x^4 - 5x^3 + 5x^2 + kx - 6$  is exactly divisible by  $x - 2$ , what is the value of  $k$ ? 29\_\_\_\_\_

30. If 3 coins are tossed, what is the probability that exactly 2 of them will come up heads? 30\_\_\_\_\_

31. Find to the nearest tenth:  $\log_2 2$  31\_\_\_\_\_

32. Solve for  $x$  in terms of  $a$ :  $\log 3a^4 - \log 2a = \log x$  32\_\_\_\_\_

33. If the graphs of the equations  $x^2 + y^2 = 1$  and  $xy = 2$  are drawn on the same set of axes, how many points will they have in common? 33\_\_\_\_\_

34. A root of  $x^3 - 6x^2 + 5x - 3 = 0$  lies between 5.1 and 5.2. Find this root to the *nearest tenth*. 34. \_\_\_\_\_

35. Find the abscissa of the point of inflection of the graph of  $y = x^3 - 3x^2 + 7x - 2$ . 35. \_\_\_\_\_

\*36. Write in determinant form an equation of the line which passes through the points (2, -1) and (3, 9). 36. \_\_\_\_\_

\*37. If  $\frac{1}{2} \begin{vmatrix} 0 & 3 & 1 \\ 0 & -2 & 1 \\ a & 5 & 1 \end{vmatrix} = 20$ , find the value of  $a$ . 37. \_\_\_\_\_

*Directions (38-40):* Write on the line at the right of *each* of the following the number preceding the expression that best completes the statement.

38. If  $(x^2 - x - 6) < 0$ , then  
 (1)  $-3 < x < -2$       (3)  $-2 < x < 3$   
 (2)  $1 < x < 6$       (4)  $-3 < x < 2$       38. \_\_\_\_\_

39. If  $M^{x-1} = N^x$ , then  $x$  equals  
 (1)  $\frac{\log N}{\log M - \log N}$       (3)  $\frac{\log M - \log N}{\log M}$   
 (2)  $\frac{\log M}{\log M - \log N}$       (4)  $\frac{\log N - \log M}{\log N}$   
 39. \_\_\_\_\_

40. The expression  $i^{88} - i^{22}$  is equal to (1) 0      (2) 2  
 (3)  $1 - i$       (4)  $1 + i$       40. \_\_\_\_\_

### Part III

*Answer ten questions from this part. Each correct answer will receive  $2\frac{1}{2}$  credits. No partial credit will be allowed. Questions marked \* are based upon optional topics in the syllabus. Write your answer on the line at the right.*

41. One root of the equation  $2x^3 + 7x^2 + 6x = 5$  is  $\frac{1}{2}$ . Write a quadratic equation which can be used to find the other two roots 41. \_\_\_\_\_

42. A man piles 150 posts in layers so that each layer has one less post than the layer below. If the top layer has 3 posts, find the number of layers. 42. \_\_\_\_\_

43. Solve for  $x$ :  $3x - (7 + 6x) \geq 17 - x$ . 43. \_\_\_\_\_

44. Find the coordinates of the center of the circle whose equation is  $x^2 - 14x + y^2 + 6y + 54 = 0$ . 44. \_\_\_\_\_

45. Express the complex number  $6(\cos 30^\circ + i \sin 30^\circ)$  in  $a + bi$  form. 45. \_\_\_\_\_

46. Find the modulus of the complex number  $-1 + 2i$ . 46. \_\_\_\_\_

47. The acceleration due to the earth's gravitational attraction varies inversely as the square of the distance from the center of the earth. The acceleration is 32 feet per second per second at 4,000 miles from the center. What is the acceleration in feet per second per second on a satellite 8,000 miles from the center of the earth? 47\_\_\_\_\_
48. If  $f(u) = 3u + 1$  and  $g(v) = v^2 - 1$ , find  $f(3) + g(0)$ . 48\_\_\_\_\_
49. How many distinct points does the graph of  $y = (x - 1)^2(x - 2)$  have in common with the  $x$ -axis? 49\_\_\_\_\_
50. In how many distinct ways may 2 red flags, 2 white flags and 3 blue flags be arranged on a pole if all 7 flags are used in each arrangement? 50\_\_\_\_\_
51. A body moves along a line in such a way that its distance  $S$  in feet from a fixed point on the line at the end of  $t$  seconds is given by the equation:  $S = 3t^3 + 2t^2 + 3$ . Find the acceleration in feet per second per second when  $t = 2$ . 51\_\_\_\_\_
52. Given the complex number  $N = 8(\cos 60^\circ + i \sin 60^\circ)$ . Express in polar form that cube root of  $N$  which, when represented graphically, lies in the *second* quadrant. 52\_\_\_\_\_
53. Write an equation of the straight line tangent to the curve  $y = 2x^3 + 6$  at the point  $(-1, 4)$ . 53\_\_\_\_\_
- \*54. Transform  $x^2 + y^2 - 4x + 3 = 0$  into an equation in polar coordinates. 54\_\_\_\_\_
- \*55. Find the radius of the circle whose equation is  $r = \sin \theta$ . 55\_\_\_\_\_

# FOR TEACHERS ONLY

## 12A

### INSTRUCTIONS FOR RATING TWELFTH YEAR MATHEMATICS 12A (Advanced Algebra)

Wednesday, January 25, 1961 — 1:15 to 4:15 p.m., only

Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

#### Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 23-25, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3 or 4.

- |   |                     |
|---|---------------------|
| (1) 4   | (12) $8\frac{3}{4}$ |
| (2) $m^2 + 4m$  | (13) 0              |
| (3) $\frac{7}{12}$  | (14) 1              |
| (4) $(x + 3)^2 + (y - 4)^2 = 25$<br>or<br>$x^2 + y^2 + 6x - 8y = 0$ | (15) 1 and 2        |
| (5) 1.06  | (16) 2              |
| (6) $\frac{7}{30}$  | (17) $-\frac{3}{4}$ |
| (7) -3  | (18) 13             |
| (8) $\sqrt{34}$   | (19) $45ax^4$       |
| (9) $x + y = k$   | (20) $\frac{15}{4}$ |
| (10) (9, 0)   | (21) $\frac{3}{10}$ |
| (11) $\frac{4}{3}$  | (22) 16             |
|   | (23) 1              |
|   | (24) 4              |
|   | (25) 3              |

## Part II

Allow  $2\frac{1}{2}$  credits for each of not more than 10 correct answers; allow no partial credit. If more than ten questions have been answered, only the first ten of these should be considered. For questions 38–40, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3 or 4.

(26)  $\frac{1}{3}$

(27) 5

(28) 1

(29) 5

(30)  $\frac{3}{8}$

(31) 0.4

(32)  $\frac{3a^3}{2}$

(33) none

(34) 5.1

(35) 1

(36)  $\begin{vmatrix} x & y & 1 \\ 2 & -1 & 1 \\ 3 & 0 & 1 \end{vmatrix} = 0$

(37) 8

(38) 3

(39) 2

(40) 2

## Part III

Allow  $2\frac{1}{2}$  credits for each of not more than 10 correct answers; allow no partial credit. If more than ten questions have been answered, only the first ten of these should be considered.

(41)  $2x^2 + 8x + 10 = 0$

or

$x^2 + 4x + 5 = 0$

(42) 15

(43)  $x \leq -12$

(44) (7, -3)

(45)  $3\sqrt{3} + 3i$

(46)  $\sqrt{5}$

(47) 8

(48) 9

(49) 2

(50) 210

(51) 40

(52)  $2(\cos 140^\circ + i \sin 140^\circ)$

(53)  $6x - y + 10 = 0$

(54)  $r^2 - 4r \cos \theta + 3 = 0$

(55)  $\frac{1}{2}$