Reviewed by: .....

Percent: .....

The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

ADVANCED ALGEBRA

Wednesday, January 21, 1959 — 9:15 a.m. to 12:15 p.m., only

Name of pupil..... Name of school.....

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answer on the line at the right.

1 If $(3 - 2i)(2 + i)$ is expressed in the form $a + bi$, in which a and b are real numbers, find the value of a. 1.....

2 Write an equation of the line which passes through the point $(-3, 2)$ and is parallel to the line whose equation is $3y = 2x + 5$. 2.....

3 For what value of k will the graph of $y = 3x^2 + 2x + k$ be tangent to the x-axis? 3.....

4 Find the value of m for which $(x - 2)$ is a factor of $x^4 - x^3 + mx^2 - 2x + 12$. 4.....

5 Solve the equation $x^{\frac{3}{2}} = 64$ for the real value of x. 5.....

6 Solve the equation $4^{2x-3} = \frac{1}{16}$ for x. 6.....

7 Given $\log N = 9.5353 - 10$. Find $\log \sqrt[3]{N}$. 7.....

8 If $\log 2 = x$ and $\log 3 = y$, express $\log 12$ in terms of x and y. 8.....

9 Express in simplest form: $\dfrac{1 + \dfrac{2}{x - 1}}{2 - \dfrac{x - 3}{x - 1}}$ 9.....

10 Two roots of the equation $x^3 + px + q = 0$ are 1 and -3. Find the third root. 10.....

[OVER]

11 Find the product of the roots of the equation $2x^3 - 4x^2 + x + 8 = 0$. 11.....

12 If $f(x) = x^2 + 2x$, find $f(a - 2)$. 12.....

13 Find the value of n when ${}_nC_3 = 45$. 13.....

14 How many different amounts of money can be made from a penny, a nickel, a dime and a quarter, taking one or more coins at a time? 14.....

15 In how many different ways may 3 students be seated in a row of 5 chairs in a classroom? 15.....

16 A committee of 3 is to be selected from a group of 10 people. What is the probability that a particular person of the group will be a member of the committee? 16.....

17 Write in *simplest* form the 5th term in the expansion of $\left(x + \frac{1}{x}\right)^8$. 17.....

18 Find the value of $3x^0 + \left(\frac{2}{x}\right)^{-2}$ when $x = 4$. 18.....

19 Solve the following set of equations for x:

$$\frac{1}{x} + \frac{2}{y} = 3$$

$$\frac{5}{x} - \frac{2}{y} = 9$$

19.....

20 Using k as the constant of variation, write an equation that represents the relationship: x varies directly as the square of y and inversely as z 20.....

21 If the equation of the axis of symmetry of $2x^2 - px + 7 = 0$ is $x = 3$, find the value of p. 21.....

22 Find the slope of the line that passes through the points $(-2, 4)$ and $(6, -2)$. 22.....

23 Given: $y = \frac{x - 2}{x + 1}$. Express x as a function of y. 23.....

24 Find the sum of the infinite geometric progression $9, -3, 1, \ldots$. 24.....

25 [Write the *letter* preceding the correct answer in the space provided.] The equation $\sqrt{x^2 - 12} + 2 = 0$ has (*a*) one positive root as its only root (*b*) one negative root as its only root (*c*) one positive and one negative root (*d*) no roots 25.....

Part II

Answer ten questions from this part. Each correct answer will receive 2½ credits. No partial credit will be allowed. Questions marked * are based upon optional topics in the syllabus. Write your answer on the line at the right.

26 Find the number of negative roots of the equation $x^4 + x^3 + x - 1 = 0$. 26.....

27 Write an equation whose roots are the roots of the equation $x^3 + x + 1 = 0$ each decreased by 2. 27.....

28 Write an equation whose roots are the roots of the equation $x^3 + 3x^2 + 2x + 1 = 0$ each multiplied by 2. 28.....

29 Find the integral root of the equation $x^3 - 6x^2 + 13x - 10 = 0$. 29.....

30 Find all the roots of the equation $x^4 - 3x^2 - 4 = 0$. 30.....

31 Find the nonintegral root of the equation $2x^3 - 3x^2 - 3x + 2 = 0$. 31.....

32 Solve the equation $x^2 + 2kx + k^2 - 4 = 0$ for x in terms of k. 32.....

33 Between what two consecutive integers does the real root of the equation $x^3 + x + 3 = 0$ lie? 33.....

34 One root of the equation $3x^3 + x^2 + 5x - 2 = 0$ lies between 0.3 and 0.4. Find this root to the *nearest tenth*. 34.....

35 When $f(x)$ is divided by $x + 3$, the quotient is $x^3 + x + 1$ and the remainder is 5. Find $f(x)$. 35.....

36 Combine into a single term $\sqrt{-72} + \sqrt{-\frac{9}{8}}$. 36.....

37 [Write the *letter* preceding the correct answer in the space provided.] The roots of the equation $x^3 - 6x^2 + 14x - 12 = 0$ are (*a*) 1, 2, 3 (*b*) $-1, -2, -3$ (*c*) $2, 2 \pm i$ (*d*) $2, 2 \pm i\sqrt{2}$ 37.....

*38 A tangent to the curve $y = x^2 - 4x + 10$ has a slope of 2. Find the coordinates of the point at which the tangent touches the curve. 38.....

*39 Find the abscissa of the point of inflection of the graph of $y = x^3 - 3x^2 - 2x + 1$. 39.....

*40 A body moves along a line in such a way that its distance S in feet from a fixed point on the line at the end of t seconds is given by the equation $S = t^2 - 6t + 14$. Find the velocity of the body in feet per second when $t = 8$. 40.....

Part III

Answer ten questions from this part. Each correct answer will receive 2½ credits. No partial credit will be allowed. Questions marked * are based upon optional topics in the syllabus. Write your answer on the line at the right.

41 The first three terms of a geometric progression are $(x - 5)$, $(x + 1)$ and $(2x + 11)$. Find the positive value of x. 41.....

42 If $\log a = 0.6202$, find the value of $a^{1.3}$ to the *nearest hundredth*. 42.....

43 Find, to the *nearest tenth*, the value of x in the equation $5^x = 14.9$. 43.....

44 Given $A = P(1 + r)^{10}$. Express $\log (1 + r)$ in terms of $\log A$ and $\log P$. 44.....

45 [Write the letter preceding the correct answer in the space provided.] The graphs of the equations $y = x^2 - x - 2$ and $y = 2x^2 - 2x - 4$ are drawn on the same set of axes. The graphs (*a*) coincide (*b*) have the same y-intercept (*c*) have the same x-intercepts (*d*) do not intersect 45.....

46 [Write the letter preceding the correct answer in the space provided.] The graph of $y = 2^x$ lies in (*a*) the first quadrant only (*b*) the first and second quadrants (*c*) the first and third quadrants (*d*) the first and fourth quadrants 46.....

47 Given: Two parallel lines drawn on the same set of axes. Points $(1, -2)$ and $(2, 2)$ are on one line and $(4, y)$ and $(7, 8)$ are on the other line. Find the value of y. 47.....

48 The graphs of $2x + 3y + b = 0$ and $(x - 3)^2 + y^2 = 16$ are drawn on the same set of axes. Find the value of b if the graph of the straight line passes through the center of the circle. 48.....

49 A man bought $4n$ articles for c cents each. When he had sold $3n$ of the articles, he found he had recovered his initial investment. Express in terms of c the amount he charged for each article. 49.....

50 Two points, C and D, are m miles apart. Tom leaves C for D, traveling at a uniform rate of p miles per hour, and Harry leaves D for C, traveling at a uniform rate of b miles per hour. If they start at the same time and meet at the end of h hours, express p in terms of h, m and b. 50.....

51 One man can do a job in x days and another man can do the job in y days. If both men work together, what part of the job will be finished at the end of 5 days? [It is assumed that x and y are sufficiently large that the job will not be finished in less than 5 days.] 51.....

52 If $f(x)$ is a polynomial with real coefficients and if $a + bi$ is a root of $f(x) = 0$, what other complex number must also be a root of this equation? 52.....

*53 Express $1 - i$ in the form $r(\cos \theta + i \sin \theta)$. 53.....

*54 Express $2(\cos 120° + i \sin 120°)$ in the form $a + bi$. 54.....

*55 Express *one* of the roots of $x^3 + 27 = 0$ in the form $r(\cos \theta + i \sin \theta)$. 55.....

I have studied advanced algebra for .... weeks and have had .... recitations per week.

I do so declare.....
(*Signature*)

FOR TEACHERS ONLY

AA

INSTRUCTIONS FOR RATING
ADVANCED ALGEBRA

Wednesday, January 21, 1959 — 9:15 a.m. to 12:15 p.m., only

Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use check marks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For question 25, allow credit if the pupil has written the correct answer instead of the letter *d*.

(1) 8 (Allow credit for $8 - i$)

(2) $3y = 2x + 12$

(3) $\frac{1}{3}$

(4) -4

(5) 16

(6) $\frac{1}{2}$

(7) 9.8451–10 *or* $\bar{1}.8451$ *or* $-.1549$

(8) $2x + y$

(9) 1

(10) 2

(11) -4

(12) $a^2 - 2a$

(13) 10

(14) 15

(15) 60

(16) $\frac{3}{10}$

(17) 70

(18) 7

(19) $\frac{1}{2}$

(20) $x = \frac{ky^2}{z}$

(21) 12

(22) $-\frac{3}{4}$

(23) $\frac{-y-2}{y-1}$ *or* $\frac{2+y}{1-y}$

(24) $\frac{27}{4}$

(25) *d*

[OVER]

Part II

Allow 2½ credits for each of not more than ten correct answers; allow no partial credit. If more than ten questions have been answered, only the first ten of these should be considered. Do not allow credit if answers to questions 27 and 28 are not expressed as equations.

(26) one

(27) $y^3 + 6y^2 + 13y + 11 = 0$

(28) $y^3 + 6y^2 + 8y + 8 = 0$

(29) 2

(30) ± 2 and $\pm i$

(31) $\frac{1}{2}$

(32) $-k \pm 2$

(33) -2 and -1

(34) 0.3

(35) $x^4 + 3x^3 + x^2 + 4x + 8$

(36) $6\frac{1}{4}i\sqrt{2}$ *or* $6\frac{1}{4}\sqrt{-2}$

(37) *d*

(38) (3,7)

(39) 1

(40) 10

Part III

Allow 2½ credits for each of not more than ten correct answers; allow no partial credit. If more than ten questions have been answered, only the first ten of these should be considered. For questions 45–46, allow credit if the pupil has written the correct answer instead of the letter *b* or *c*.

(41) 8

(42) 8.52

(43) 1.7

(44) $\dfrac{\log A - \log P}{10}$

(45) *c*

(46) *b*

(47) -4

(48) -6

(49) $\dfrac{4c}{3}$

(50) $\dfrac{m - bh}{h}$

(51) $5\left(\dfrac{1}{x} + \dfrac{1}{y}\right)$

(52) $a - bi$

(53) $\sqrt{2}\,(\cos 315° + i \sin 315°)$

(54) $-1 + i\sqrt{3}$

(55) $3\,(\cos 60° + i \sin 60°)$ *or* $3\,(\cos 180° + i \sin 180°)$ *or* $3(\cos 300° + i \sin 300°)$