

## ADVANCED ALGEBRA

Tuesday, January 21, 1958 — 9:15 a.m. to 12:15 p.m., only

Name of pupil.....Name of school.....

## Part I

Answer all questions in this part. Each correct answer will received 2 credits. No partial credit will be allowed. Write your answer on the line at the right.

- 1 Find the slope of the straight line that passes through the points  $(2, -3)$  and  $(7, -2)$ . 1.....
- 2 Write an equation of the line which passes through the origin and is parallel to  $x - y + 7 = 0$ . 2.....
- 3 If  $f(x) = 2x - 1$ , find  $f(a + 1)$ . 3.....
- 4 Solve for  $x$ :  $\frac{x-2}{x+4} = \frac{x+2}{x-4}$  4.....
- 5 Solve for  $x$ :  $a^{x-2} = \left(\frac{1}{a}\right)^x$  5.....
- 6 If  $x^2 - t^2 = 0$ , express  $x$  in terms of  $t$ . 6.....
- 7 Find the *two* values of  $k$  for which the equation  $kx^2 + 2x + k = 0$  will have equal roots. 7.....
- 8 Express  $\frac{4i}{1+i}$  in the form  $a + bi$ . 8.....
- 9 Write in simplest form the fourth term *only* in the expansion of  $(a^2 + a^{-2})^6$ . 9.....
- 10 The quantity  $A$  varies as the product of  $\theta$  and the square of  $r$ . If  $A = \frac{3}{4}$  when  $\theta = \frac{1}{6}$  and  $r = 3$ , express  $A$  in terms of  $\theta$  and  $r$ . 10.....
- 11 Find the  $x$ -intercept of the line whose equation is  $2x - y - 3 = 0$ . 11.....
- 12 Find the coordinates of the point of intersection of the lines  $x + 3 = 0$  and  $y - 7 = 0$ . 12.....
- 13 The equation of the axis of symmetry of the parabola  $y = x^2 + kx$  is  $x = 4$ . Find the value of  $k$ . 13.....
- 14 If the graphs of  $x^2 - y^2 = 9$  and  $y = 2$  are drawn on the same set of axes, how many points do the graphs have in common? 14.....
- 15 Find the remainder when  $x^{25} + 2$  is divided by  $x - 1$ . 15.....

- 16 Find the ninth term of the arithmetic progression  $-6, -\frac{9}{2}, -3, \dots$  16.....
- 17 Find the value of  ${}_{20}C_{13}$ . 17.....
- 18 An automobile manufacturer offers cars having three choices of body design, eight color combinations and two different engines. Body design, color and engine are available in any combination. To assure immediate delivery of any order, what is the *least* number of automobiles a dealer must have in stock? 18.....
- 19 In a league of six high school basketball teams, how many league games will be played in a season if each team plays two games with every other team? 19.....
- 20 A two-digit number whose digits may be repeated is formed from the digits 2, 3, 4, 5, 6, 7, 8. What is the probability that it is an even number? 20.....
- 21 Given  $\log 3.46 = 0.5391$ , find  $\log \sqrt[3]{3460}$ . 21.....
- 22 Express in *simplest form*:  $\frac{2(2x+3)}{3x-2} + 3$  22.....  
 $\frac{3(2x+3)}{3x-2} - 2$

*Directions (23-26):* For each of the following, tell whether the statement is always true, sometimes true or never true by writing the word *always*, *sometimes* or *never* on the line at the right.

- 23 If  $K$  represents the same positive number in both of the equations  $y - 2x = K$  and  $4x - 2y = K$ , the graphs of the equations intersect. 23.....
- 24 The sum of the roots of the equation  $ax^2 + bx + c = 0$  (where  $a$  is not equal to zero) is equal to their product when  $b = -c$ . 24.....
- 25 If an equation of the form  $y = x^3 + ax^2 + bx + c$  has  $y$  positive when  $x = 2$  and  $y$  negative when  $x = 3$ , then  $y = 0$  for at least one value of  $x$  between  $x = 2$  and  $x = 3$ . 25.....
- 26 The product of  $3 + ai$  and  $3 - ai$ , where  $a$  represents a real number, is a positive number. 26.....

*Directions (27-30):* Indicate the correct completion for each of the following by writing the letter  $a$ ,  $b$ ,  $c$  or  $d$  on the line at the right.

- 27 The number  $3.2727 \dots$ , where the digits 2 and 7 repeat infinitely as indicated, is equal to (a) 3.273 (b)  $\frac{33}{10}$  (c)  $\frac{36}{11}$  (d)  $\frac{109}{33}$  27.....
- 28 The expression  $\log \left( \frac{x}{y} \right)^n$  is identical with (a)  $n(\log x - \log y)$   
 (b)  $\log nx - \log ny$  (c)  $\frac{\log x}{\log y}$  (d)  $\frac{\log nx}{\log ny}$  28.....
- 29 If three of the roots of an equation with real and rational coefficients are  $2$ ,  $2i$  and  $2 + 2i$ , the lowest possible degree of the equation is (a) 6  
 (b) 5 (c) 3 (d) 4 29.....
- 30 The equation  $\sqrt{x+2} = x$  has (a) no root (b)  $-1$  as its only root  
 (c)  $2$  as its only root (d) the two roots,  $2$  and  $-1$  30.....



Part II

Answer ten questions from this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Questions marked \* are based upon optional topics in the syllabus. Write your answer on the line at the right.

- 31 How many imaginary roots has the equation  $x^2 + 2x + 3 = 0$ ? 31.....
- 32 Write an equation whose roots are those of the equation  $x^2 + 2x + 3 = 0$  each multiplied by 2. 32.....
- 33 Write an equation whose roots are those of the equation  $x^2 + 3x^2 - x + 3 = 0$  each increased by 2. 33.....
- 34 Find an integral root of the equation  $x^4 - x^3 - 9x^2 - 10x - 8 = 0$ . 34.....
- 35 Find a nonintegral fractional root of the equation  $2x^4 - x^2 - 4x^2 + 10x - 4 = 0$ . 35.....
- 36 The smallest positive root of  $x^3 - 8x^2 + 19x - 13 = 0$  lies between which two successive integers? 36.....
- 37 A root of  $x^2 + 2x^2 - 5x - 2 = 0$  lies between  $x = 1$  and  $x = 2$ . Find this root to the nearest integer. 37.....
- 38 Write an equation with real integral coefficients and of lowest possible degree, whose roots are 2,  $i$  and  $-i$ . 38.....
- 39 Find all of the roots of the equation  $x^4 - 10x^2 + 9 = 0$ . 39.....
- 40 Find the quotient when  $2x^4 + 7x^3 - x^2 - 15x - 9$  is divided by  $x + 3$ . 40.....
- 41 Expand and express the result in simplest form:  $(\sqrt{x} + \sqrt{y})^2$  41.....
- 42 Solve the following system of equations for  $x$ :  

$$\frac{3}{x} - \frac{2}{y} = 2$$

$$\frac{6}{x} + \frac{2}{y} = 1$$
 42.....
- \*43 Find the abscissa of the maximum point of the graph of  $y = 2x^3 + 3x^2 - 12x - 20$ . 43.....
- \*44 Find the abscissa of the point of inflection of the graph of  $y = x^3 - 2x^2 - x + 1$ . 44.....
- \*45 Find the slope of the tangent to the graph of  $y = -3x^3 + 12x^2$  at the point (2, 24). 45.....

143

## Part III

Answer ten questions from this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Questions marked \* are based upon optional topics in the syllabus. Write your answer on the line at the right.

- 46 Find to the nearest tenth the value of  $2^{3.4}$ . 46.....
- 47 Find to the nearest tenth the value of  $\log_2 5$ . 47.....
- 48 Find the value of  $x$  if  $x = 10^{\log 7}$ . 48.....
- 49 Given  $\log_b 2 = 0.693$  and  $\log_b 3 = 1.099$ , find  $\log_b 12$ . 49.....

Directions (50–52): Indicate the correct completion for each of the following by writing the letter  $a$ ,  $b$ ,  $c$  or  $d$  on the line at the right.

- 50 The length of the line segment joining the points of intersection of the graphs of  $3y = 4x$  and  $x^2 + y^2 = 25$  is (a) 5 (b) 6 (c) 8 (d) 10 50.....
- 51 The graphs of  $y = 2^x$  and  $y = 3^x$  have (a) the same  $x$ -intercept (b) the same  $y$ -intercept (c) no point in common (d) two points in common 51.....
- 52 The maximum value of  $y$  on the graph of  $y = -x^2 - 2x - 3$  is (a)  $-3$  (b)  $-2$  (c)  $-1$  (d) 0 52.....
- 53 Find  $n$  if  ${}_nP_2 = 20$ . 53.....
- 54 A rectangular sheet of cardboard measures  $a$  inches by  $b$  inches. A square,  $c$  inches on a side, is cut out of each corner and the sides are turned up to form an open box. Express the number of cubic inches in the volume of the box in terms of  $a$ ,  $b$  and  $c$ . 54.....
- 55 A line whose length is  $d$  is divided into two segments such that the larger segment  $x$  is the mean proportional between the whole line and the smaller segment. Write a proportion that states the above fact. 55.....
- 56 The distance between two cities is sixty miles. If a man travels from one to the other at an average speed of 20 m.p.h. and returns at an average speed of 30 m.p.h., what is his average speed in miles per hour for the round trip? 56.....
- 57 A mixture of nuts weighing  $p$  pounds is one-fourth walnuts. If  $q$  pounds of filberts are added to the mixture, what fractional part of the new mixture is walnuts? 57.....
- \*58 Find to the nearest degree the amplitude of  $3 + 5i$ . 58.....
- \*59 Express  $4(\cos 300^\circ + i \sin 300^\circ)$  in  $a + bi$  form. 59.....
- \*60 Write in polar form one of the imaginary fourth roots of 81. 60.....



# FOR TEACHERS ONLY

# AA

## INSTRUCTIONS FOR RATING ADVANCED ALGEBRA

Tuesday, January 21, 1958 — 9:15 a.m. to 12:15 p.m., only

Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use check marks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

### Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 27–30, allow credit if the pupil has written the correct answer instead of the letter *a*, *b*, *c* or *d*.

- |                              |                    |
|------------------------------|--------------------|
| (1) $\frac{1}{3}$            | (16) 6             |
| (2) $y = x$                  | (17) 190           |
| (3) $2a + 1$                 | (18) 48            |
| (4) zero                     | (19) 30            |
| (5) 1                        | (20) $\frac{1}{4}$ |
| (6) $\pm t$                  | (21) 1.1797        |
| (7) $\pm 1$                  | (22) $x$           |
| (8) $2 + 2i$                 | (23) never         |
| (9) 20                       | (24) always        |
| (10) $\frac{1}{2}\theta r^2$ | (25) always        |
| (11) $\frac{3}{2}$           | (26) always        |
| (12) $(-3, 7)$               | (27) $c$           |
| (13) $-8$                    | (28) $a$           |
| (14) 2                       | (29) $b$           |
| (15) 3                       | (30) $c$           |

[OVER]

145

## Part II

Allow 2 credits for each of not more than ten correct answers; allow no partial credit. If more than ten questions have been answered, only the first ten of these should be considered. Do not allow credit if answers to questions 32, 33 and 38 are not expressed as equations.

(31) 2

(39)  $\pm 1, \pm 3$

(32)  $x^2 + 8x + 24 = 0$

(40)  $2x^3 + x^2 - 4x - 3$

(33)  $x^3 - 3x^2 - x + 9 = 0$

(41)  $x + 2\sqrt{xy} + y$  or  $x + 2\sqrt{x}\sqrt{y} + y$

(34) 4 or -2

(42) 3

(35)  $\frac{1}{2}$

(43) -2

(36) 1 and 2

(44)  $\frac{2}{3}$

(37) 2

(45) 12

(38)  $x^3 - 2x^2 + x - 2 = 0$

## Part III

Allow 2 credits for each of not more than ten correct answers; allow no partial credit. If more than ten questions have been answered, only the first ten of these should be considered. For questions 50-52, allow credit if the pupil has written the correct answer instead of the letter *a*, *b*, *c* or *d*.

(46) 5.3

(47) 2.3

(48) 7

(49) 2.485

(50) *d*

(51) *b*

(52) *b*

(53) 5

(54)  $c(a - 2c)(b - 2c)$  or  $abc - 2ac^2 - 2bc^2 + 4c^3$

(55)  $\frac{d}{x} = \frac{x}{d-x}$

(56) 24

(57)  $\frac{p}{4(p+q)}$

(58)  $59^\circ$

(59)  $2 - 2\sqrt{3}i$

(60)  $3(\cos 90^\circ + i \sin 90^\circ)$  or  $3(\cos 270^\circ + i \sin 270^\circ)$