ADVANCED ALGEBRA

Tuesday, January 22, 1957—9:15 a.m. to 12:15 p.m., only

Part I

Answer all questions in this part. Each correct answer will receive 2½ credits. No partial credit will be allowed.

1. Write an equation of the line that passes through the point $(1, -2)$ and that is parallel to the line whose equation is $3x + 5y = 17$. 1.....

2. In *simplest form*, express $\dfrac{2 + \sqrt{-3}}{1 + 2\sqrt{-3}}$ as an equivalent fraction with a real denominator. 2.....

3. Solve the equation $8^{x-5} = (\frac{1}{4})^3$ for x. 3.....

4. For what value of k is $(x + 1)$ a factor of $3x^4 - 5x^3 + kx^2 - 6x - 10$? 4.....

5. Find the cube root of 0.0753 to the *nearest thousandth*. 5.....

6. Find $\log_3 7.2$ to the *nearest tenth*. 6.....

7. Write in *simplest form* the sixth term in the expansion of $(2 + \dfrac{x}{2})^8$. 7.....

8. An equation with real coefficients has $1 - i$ and i among its roots. What is the lowest possible degree of the equation? 8.....

9. If the graphs of $x^2 - y^2 = 1$ and $4x^2 + y^2 = 16$ are drawn on the same set of axes, how many points will they have in common? 9.....

10. How many different committees of 3 can be chosen from 10 people if a certain person must always be included in each committee? 10.....

11. If an event may happen in 4 ways and fail to happen in 7 ways, each equally probable, what is the probability that the event will happen? 11.....

12. If $\log_a 2 = x$ and $\log_a 5 = y$, express $\log_a 40$ in terms of x and y. 12.....

13. The sum of two roots of the equation $2x^3 - 6x^2 - 8x + q = 0$ is zero. Find the third root. 13.....

14. Find the rational fractional root of $2x^3 + x^2 + 2x - 12 = 0$. 14.....

15. If x varies directly as y and inversely as the square of z, and if $x = 72$ when $y = 3$ and $z = 1/2$, find x when $y = 1/4$ and $z = 1/6$. 15.....

16. If $f(x) = \dfrac{x^{\frac{2}{3}} + x^{-1}}{\quad}$, find in *simplest form* the value of $f(8)$. 16.....

17. For what positive value of k is the graph of $y = kx^2 + 6x + k + 8$ tangent to the x-axis? 17.....

18. Find the number of real roots of the equation $x^5 + x^3 + 1 = 0$. 18.....

19. Transform the equation $x^4 - 27x^2 - 27x - 162 = 0$ into an equation whose roots are those of the original equation each divided by 3. 19.....

20. Transform the equation $2x^3 - x^2 + 2x + 7 = 0$ into an equation whose roots are those of the original equation each decreased by 2. 20.....

Part II

Answer five questions from this part. Show all work.

21. Solve the equation $2x^4 + x^3 - 3x^2 + 5x - 2 = 0$. [10]

22. Find to the *nearest tenth* the real root of $2x^3 - 5x^2 - 2 = 0$. [10]

23. *a.* *Draw* the graph of $y = 2^x$ for values of x from -2 to 4, inclusive. [6]

b. On the same set of axes used in part *a*, *draw* the graph of $y = x^2 - 4x + 2$ for values of x from -1 to 5, inclusive. [3]

c. From the graphs made in answer to parts *a* and *b*, estimate to *tenths* the real root of the equation $2^x = x^2 - 4x + 2$. [1]

24. Given $A^{0.3} = \dfrac{h}{T^{0.7}}$. If $A = 4.91$ and $h = 2.86$, find T to the *nearest hundredth*. [10]

25. [The proof of this formula is *not* a required topic in the syllabus for Twelfth Year Mathematics.]

a. Prove that ${}_nC_r$ equals ${}_nC_{n-r}$. [8]

b. Find the value of ${}_{40}C_{37}$. [2]

26. A man traveled 60 miles by bus and 600 miles by plane, taking 6 hours for the trip. On the return trip the average speed of the plane was reduced by 50 miles an hour, but the speed of the bus was increased by 10 miles an hour so that the return trip also took 6 hours. Find the average speed of the plane and the average speed of the bus. [5, 5]

27. Bruce and Roy working together can do a job in 6 hours, Bruce and John working together can do the job in 8 hours and Roy and John together can do the job in 12 hours. How many hours will it take *each* boy working alone to do the job? [4, 6]

*28. *a.* Using derivatives, find the coordinates of the maximum point, the minimum point and the point of inflection of the equation $y = 2x^3 + 3x^2 - 12x + 1$. [3, 2, 3]

b. *Sketch* the graph of the equation $y = 2x^3 + 3x^2 - 12x + 1$. [2]

*29. *a.* Express $-3i$ in the form $r(\cos\theta + i\sin\theta)$. [2]

b. Express $-1 + i\sqrt{3}$ in the form $r(\cos\theta + i\sin\theta)$. [2]

c. Express $10(\cos 234° + i\sin 234°)$ in $a + bi$ form. Express the values of a and b to the *nearest tenth*. [3]

d. Express in the form $r(\cos\theta + i\sin\theta)$ the root of $x^5 - 1 = 0$ that, when represented graphically, lies in the fourth quadrant. [3]

*These questions are based upon optional topics in the syllabus.

The University of the State of New York

329TH HIGH SCHOOL EXAMINATION

TWELFTH YEAR MATHEMATICS

12A (Advanced Algebra)

Tuesday, January 22, 1957 — 9:15 a.m. to 12:15 p.m., only

Note to teacher: These questions may be used in conjunction with the regular Regents examination in advanced algebra by those pupils who have followed the outline in the twelfth year syllabus. A copy of this sheet should be distributed to each pupil qualified, together with a copy of the regular examination paper in advanced algebra. If sufficient copies of this sheet are not available, these questions may be written on the blackboard.

Part I

Directions: Since questions 18, 19 and 20 on the examination in advanced algebra are not based on topics in the twelfth year syllabus, you may replace one or more of those by any of the following questions. Indicate any substitutions by labeling the answers *A, B* or *C*. [Write answers on the regular question paper opposite the questions you are replacing.]

A Write an equation of the line that passes through the point $(1, -2)$ and that is perpendicular to the line whose equation is $2x - 3y = 10$.

B Write an equation of the circle whose center is $(1, 3)$ and whose radius is 4.

C Solve the inequality $2x - \frac{3x}{4} < 5$.

Part II

Directions: The following questions are based upon optional topics of the twelfth year syllabus. Either 30 or 31, or *both*, may be used in place of *any one or two* of the questions on part II of the examination in advanced algebra.

30 *a* Transform the equation $\sin 2\theta = \frac{12}{r^2}$ from polar coordinates to rectangular coordinates. [6]

b Which *two* of the following points lie on the graph of the equation $\sin 2\theta = \frac{12}{r^2}$? [4]

(1) $(2\sqrt{3}, 45°)$ (2) $(2\sqrt{3}, 135°)$ (3) $-2\sqrt{3}, -45°)$ (4) $(2\sqrt{6}, 195°)$

31 Using *determinants*, solve the following system of equations for x: [10]

$$2x - 3y + z = -1$$
$$3x + y - 3z = -3$$
$$-5x + 4y + 2z = 10$$

FOR TEACHERS ONLY

AA

INSTRUCTIONS FOR RATING
ADVANCED ALGEBRA
and
TWELFTH YEAR MATHEMATICS
12A (Advanced Algebra)

Tuesday, January 22, 1957 — 9:15 a.m. to 12:15 p.m., only

Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use check marks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow $2\frac{1}{2}$ credits for each correct answer; allow no partial credit. Do not allow credit if answers to questions 19 and 20 are not expressed as equations.

(1) $3x + 5y = -7$

(2) $\frac{8 - 3i\sqrt{3}}{13}$

(3) 3

(4) -4

(5) 0.422

(6) 1.8

(7) $14x^5$

(8) 4

(9) 4

(10) 36

(11) $\frac{4}{11}$

(12) $3x + y$

(13) 3

(14) $\frac{3}{2}$

(15) 54

(16) $\frac{33}{16}$

(17) 1

(18) 1

(19) $x^4 - 3x^2 - x - 2 = 0$

(20) $2x^3 + 11x^2 + 22x + 23 = 0$

Twelfth Year Mathematics (Advanced Algebra)

A $2y + 3x + 1 = 0$
B $(x - 1)^2 + (y - 3)^2 = 16$
C $x < 4$