ADVANCED ALGEBRA

Wednesday, January 25, 1956—9.15 a.m. to 12.15 p.m., only

Part I

Answer all questions in this part. Each correct answer will receive 2½ credits. No partial credit will be allowed.

1. If $f(x) = \dfrac{2x}{3x-h}$, find $f(h)$. 1.....

2. Write an equation of the line passing through the point (2,0) and parallel to the line $2x - y + 3 = 0$. 2.....

3. Write in simplest form the *fifth* term in the expansion of $(x - \sqrt{y})^6$. 3.....

4. Find the remainder when $x^{16} + 5$ is divided by $x + 1$. 4.....

5. Express in simplest form the product of $3 + 6i$ and $2 + i$. 5.....

6. Express the repeating decimal 0.0909 . . . as a common fraction. 6.....

7. A graph whose equation may be written in the form $y = x^2 + px + q$ has an axis of symmetry whose equation is $x = -2$. Find the value of p. 7.....

8. In the equation $2x^2 - 20x + 40 + k = 0$, find the value of k so that the roots shall be equal. 8.....

9. If two roots of the equation $x^3 + ax^2 + bx + c = 0$ (in which a, b and c represent integers) are 1 and $2 - 3i$, find the value of a. 9.....

10. If the graphs of $y^2 - x^2 = 4$ and $x = 2$ are drawn on the same set of axes, how many points do the graphs have in common? 10.....

11. How many different three-digit numbers greater than 300 can be made using the digits 1, 2, 3 and 4 if no digits are repeated? 11.....

12. Solve the equation $\dfrac{K + y}{K - y} = K$ for y in terms of K. 12.....

Directions (13-17): Indicate the correct completion for *each* of the following by writing on the line at the right the letter *a*, *b* or *c*.

13. The fraction $\dfrac{\log x^n}{\log y^n}$ is equal to (*a*) $\dfrac{\log nx}{\log ny}$ (*b*) $\dfrac{\log x}{\log y}$ (*c*) $\log \left(\dfrac{x}{y}\right)^n$ 13.....

14. If r varies directly as s and inversely as t, an equation representing this relationship, using k as the constant of variation, is (*a*) $rt = ks$ (*b*) $r = kst$ (*c*) $rs = kt$ 14.....

15. If the probability that an event will happen is $\frac{1}{x}$, then the probability that the event will *not* happen is (*a*) $\frac{1-x}{x}$ (*b*) $\frac{x-1}{x}$ (*c*) $\frac{1+x}{x}$ 15.....

16. When n is *not* equal to 1, ${}_nC_n$ is (*a*) less than ${}_nC_{n-1}$ (*b*) equal to ${}_nC_{n-1}$ (*c*) greater than ${}_nC_{n-1}$ 16.....

17. The equation $\sqrt{x+6} = -x$ has (*a*) no roots (*b*) one root (*c*) two roots 17.....

18. Transform the equation $x^3 - 2x^2 - x + 1 = 0$, whose roots are a, b and c, into an equation whose roots are $3a$, $3b$ and $3c$. 18.....

19. Transform the equation $x^3 - 2x^2 - x + 1 = 0$ into an equation whose roots are those of the original equation increased by 1. 19.....

20. How many imaginary roots has the equation $x^5 + x^3 + 1 = 0$? 20.....

Part II

Answer five questions from this part. Show all work.

21. Solve the equation $2x^4 - 5x^3 + 4x^2 - 5x + 2 = 0$. [10]

22. Find to the *nearest tenth* the larger of the two positive roots of $x^3 + 3x^2 - 13x + 1 = 0$. [10]

23. *a.* State and prove the Remainder Theorem. [1, 5]
b. State and prove the Factor Theorem. [1, 3]

24. *a.* If $7.6^x = 2.1$, find the value of x to the *nearest hundredth*. [4]
b. If $y = 0.49^{-1.9}$, find the value of y to the *nearest tenth*. [6]

25. *a.* Draw the graph of $y = x^3 - 3x^2$ for the interval from $x = -1$ to $x = 3$. [4]
b. On the same set of axes used in part *a*, draw the graph of $y = -x^2 + 4x - 4$ for the interval from $x = 0$ to $x = 4$. [4]
c. The axis of symmetry of the graph of $y = -x^2 + 4x - 4$ intersects the graph of $y = x^3 - 3x^2$ at the point P. Using the graphs drawn in parts *a* and *b*, find the coordinates of P. [2]

26. In a geometric progression of four terms, the third term exceeds the first by 42 and the fourth term exceeds the second by 56. Find the four terms of the progression. [10]

27. A gardener can cultivate his large vegetable garden in 1 hour and 30 minutes less time with a power cultivator than with a hand cultivator. One day his power cultivator broke down 25 minutes after he started to use it, and he required 1 hour and 50 minutes to finish cultivating his garden with the hand cultivator. How long would it have taken him to cultivate the garden using only the power cultivator? [6, 4]

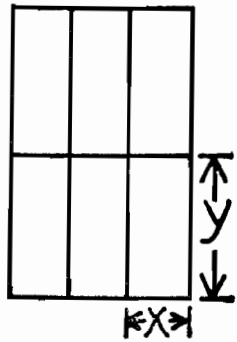
*28. *a.* Which of the numbers -3, $4i$, $-3 + 4i$ has the smallest modulus? [2]

b. Which number in part *a* has the smallest amplitude (angle)? [2]

c. Express $3 - 3i$ in polar form. [3]

d. Express $2(\cos 240° + i \sin 240°)$ in $a + bi$ form. [3]

*29. A total of 360 feet of fencing is to be used to construct 6 equal rectangular enclosures, each with dimensions of x and y as shown in the figure.



a. Express y as a function of x. [2]

b. Express the area A of one of the six enclosures in terms of x. [1]

c. Using the *derivative*, find the value of x for which A is a maximum. [5]

d. Find the value of y for which A is a maximum. [2]

* These questions are based upon the optional topics in the syllabus.

The University of the State of New York

326TH HIGH SCHOOL EXAMINATION

TWELFTH YEAR MATHEMATICS

12A (Advanced Algebra)

Wednesday, January 25, 1956—9.15 a.m. to 12.15 p.m., only

Note to teacher: These questions may be used in conjunction with the regular Regents examination in advanced algebra by those pupils who have followed the outline in the twelfth year syllabus. A copy of this sheet should be distributed to each pupil qualified, together with a copy of the regular examination paper in advanced algebra. If sufficient copies of this sheet are not available, these questions may be written on the blackboard.

Part I

Directions: Since questions 18, 19 and 20 on the examination in advanced algebra are not based on topics in the twelfth year syllabus, you may replace one or more of those by any of the following questions. Indicate any substitutions by labeling the answers *A, B* or *C*. [Write answers on the regular question paper opposite the questions you are replacing.]

A The distance between the points $(4a, 0)$ and $(1, 3a)$ is $5a$. Find the value of a.

B Solve the inequality $3 - 2x \leq x + 4$.

C Express in polar form the cube root of 8 $(\cos 30° + i \sin 30°)$ whose amplitude is smallest.

Part II

Directions: The following questions are based upon optional topics of the twelfth year syllabus. Either 30 or 31, but *not* both, may be used in place of *any one* of the questions on part II of the examination in advanced algebra.

30 Triangle ABC has vertex A at the point $(3, 2)$ and vertex B at the point $(-2, 5)$. The straight line through the vertices A and C has the equation $3x - 2y = 5$ and the straight line through the vertices B and C has the equation $2x + y = 1$.

a Write in *determinant form* an equation of the straight line passing through the vertices A and B. [3]

b Find the coordinates of vertex C. [2]

c Write in *determinant form* an expression for the area of triangle ABC. [2]

d Using the expression found in part *c*, find the area of triangle ABC. [3]

31 *a* Using polar coordinates, *sketch* the graph of $r = 3$. [3]

b Using the same pole and polar axis as in part *a*, *sketch* the graph of $r = 3 \sin \theta$. [5]

c Find the polar coordinates of a point common to the graphs of $r = 3$ and $r = 3 \sin \theta$. [2]

FOR TEACHERS ONLY

AA

INSTRUCTIONS FOR RATING
ADVANCED ALGEBRA
and
TWELFTH YEAR MATHEMATICS
12A (Advanced Algebra)

Wednesday, January 25, 1956 — 9.15 a.m. to 12.15 p.m., only

Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use check marks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow $2\frac{1}{2}$ credits for each correct answer; allow no partial credit. Do not allow credit if answers to questions 18 and 19 are not expressed as equations. For questions 13–17, allow credit if the pupil has written the correct answer instead of the letter *a, b* or *c*.

(1) 1
(2) $2x - y - 4 = 0$
(3) $15x^2y^2$
(4) 6
(5) $15i$
(6) $\frac{1}{11}$
(7) 4
(8) 10
(9) —5
(10) two
(11) twelve
(12) $y = \frac{K^2 - K}{1 + K}$
(13) *b*
(14) *a*
(15) *b*
(16) *a*
(17) *b*
(18) $x^3 - 6x^2 - 9x + 27 = 0$
(19) $x^3 - 5x^2 + 6x - 1 = 0$
(20) four

Twelfth Year Mathematics (Advanced Algebra)

A $\frac{1}{8}$
B $x \geq -\frac{1}{3}$
C $2(\cos 10° + i \sin 10°)$