ADVANCED ALGEBRA

Tuesday, January 25, 1955—9.15 a.m. to 12.15 p.m., only

Part I

Answer all questions in this part. Each correct answer will receive 2½ credits. No partial credit will be allowed.

1. Express $\frac{2}{2+\sqrt{-3}}$ as an equivalent fraction with a real denominator. 1.....

2. Write an equation of the line passing through the point (2, —3) and parallel to the line whose equation is $2x - 3y = -1$. 2.....

3. Write in simplest form the fifth term in the expansion of $(\frac{1}{\sqrt{x}} + y)^8$. 3.....

4. For what value of k is $(x - 1)$ a factor of $2x^{17} + kx^{11} - 4$? 4.....

5. Solve for x: $9^{2x-2} = 3^{8x}$ 5.....

6. Find $\log_4 3$ to the *nearest tenth.* .... 6.....

7. If the graphs of $xy = 4$ and $y = x^2$ are drawn on the same set of axes, how many points do the graphs have in common? 7.....

8. How many numbers of 3 different digits can be made from the digits 0, 1, 2, 3, 4? 8.....

9. A dealer has 10 different sets of postage stamps. A boy is allowed a choice of 4 of these sets for his birthday. How many choices does he have? 9.....

10. A committee of 3 is to be chosen by lot from a group of 9 men, one of whom is Mr. Smith. What is the probability that Mr. Smith will be among those chosen? 10.....

11. Write in the form $x^3 + px^2 + qx + r = 0$ an equation with rational coefficients, two of whose roots are 2 and $1 - i$. 11.....

12. Two of the roots of $2x^3 + 6x^2 + px + q = 0$ are 1 and —2. Find the third root. 12.....

13. The electrical resistance of a cable varies directly as its length and inversely as the square of its diameter. If a cable 1000 feet long and ½ inch in diameter has a resistance of 0.08, find the resistance in a cable 1 inch in diameter and 500 feet long. 13.....

14. If $f(x) = x^{\frac{3}{2}} - 2x^0 + x^{-1}$, find $f(4)$. 14.....

15. Find the sum of the infinite progression 9, —6, 4, ... 15.....

16. For what positive value of K is the graph of $y = x^2 - 2Kx + K + 2$ tangent to the x-axis? 16.....

17. If the equation of the axis of symmetry of $y = 2x^2 + px + q$ is $x = 3$, find the value of p. 17.....

18. Find the number of imaginary roots of the equation $x^6 + x^4 + x^2 - 1 = 0$. 18.....

19. Transform the equation $x^4 - 8x^3 + 4x^2 + 16 = 0$ into an equation whose roots are those of the original equation each divided by 2. 19.....

20. Transform the equation $x^3 - x^2 - x + 1 = 0$ into an equation whose roots are those of the original equation each decreased by 3. 20.....

Part II

Answer five questions from this part. Show all work.

21. Find to the *nearest tenth* the real root of $2x^3 + x^2 + 2x - 8 = 0$ [10]

22. Solve the equation $2x^4 - 9x^3 + 11x^2 - 4x - 6 = 0$. [10]

23. Given $T = \dfrac{50d^3}{L^{1.7}}$, find L to the *nearest integer* when $d = 3.8$ and $T = 7.3$. [10]

24. *a.* Draw the graph of $y = 2^x$ from $x = -2$ to $x = 3$. [4]
b. On the same set of axes as in answer to *a*, draw the graph of $y = -3x^2 + 5x + 1$ from $x = -1$ to $x = 3$. [4]
c. From the graphs made in answer to *a* and *b* determine the number of real roots of the equation $2^x = -3x^2 + 5x + 1$. [2]

25. Given the progression: 1, $(1 + k)$, $(1 + k)^2$, ...
a. Find the sum S of n terms of this progression in terms of k and n. [5]
b. Find to the *nearest tenth* the value of S when $k = .03$ and $n = 20$. [5]

26. Two tanks contain a mixture of water and insect spray. The first tank has 15 gallons of water and 3 gallons of spray and the second has 6 gallons of water and 3 gallons of spray. How many gallons must be drawn from each tank to obtain 6 gallons of mixture that is 20% spray? [10]

27. When the rate of an auto is increased 15 miles per hour, the time that it takes its wheels to make one revolution is decreased 1/22 of a second. Find the rate of the auto if the circumference of the wheels is 6 feet. [10]

*28. A ball is thrown vertically upward with an initial velocity of 160 feet per second. The height s reached in t seconds is given by the equation $s = 160t - 16t^2$.
a. Find the velocity at the end of 2 seconds and at the end of 7 seconds. [2, 2]
b. How high will it rise? [4]
c. Find its acceleration. [2]

*29. *a.* Express $8(\cos 300° + i \sin 300°)$ in $a + bi$ form. [3]
b. Express $-2i$ in the form $r(\cos \theta + i \sin \theta)$. [2]
c. Find to the *nearest degree* the amplitude (angle) of $-2 - i$. [2]
d. Express in the form $r(\cos \theta + i \sin \theta)$ the root of $x^6 - 1 = 0$ that when represented graphically lies in the second quadrant. [3]

* This question is based upon one of the optional topics in the syllabus. Either 28 *or* 29, or *both*, may be used for a total of *five* questions to be answered from part II.