ADVANCED ALGEBRA

Tuesday, January 26, 1954—9.15 a. m. to 12.15 p. m., only

Part I

Answer all questions in this part. Each correct answer will receive 2½ credits. No partial credit will be allowed.

1. Express the product of $(3 + i\sqrt{3})$ and $(1 - i\sqrt{3})$ in the form of $a + bi$. 1.....

2. Write an equation of the line passing through the point $(0, 5)$ and parallel to the line $3x - y = 6$. 2.....

3. Write in simplest form the *fourth* term in the expansion of $(\sqrt{x} + \frac{1}{y})^7$ 3.....

4. If $f(x) = x^2 + 5x$, find $f(a - 1)$. 4.....

5. Find the remainder when $2x^3 - 4x^2 + 3$ is divided by $x + 1$. 5.....

6. If $\log_{10} 2 = a$ and $\log_{10} 3 = b$, find the value of 10^{a+b} 6.....

7. Find the value of $\sqrt[3]{0.423}$ to the *nearest hundredth*. 7.....

8. Write an equation, with rational coefficients and of lowest possible degree, two of whose roots are -1 and $2 + i$. 8.....

9. By how much does the sum of the roots of the equation $x^3 - 7x^2 + 3x - 5 = 0$ exceed their product? 9.....

10. For what *positive* value of m is the graph of $y = x^2 - mx + 3 + m$ tangent to the x-axis? 10.....

11. Find the sum of the infinite progression $5, \frac{-5}{2}, \frac{5}{4}, \ldots$ 11.....

12. Using k as the constant of variation, write an equation expressing the following relationship: The distance (D) required for an automobile to stop varies directly as the square of its speed (r) and inversely as the coefficient of friction (c) of its tires on the road. 12.....

13. If $y = \frac{2x - 1}{x - 1}$ express x in terms of y. 13.....

14. The equation $\sqrt{x+2} + x = 0$ has (a) only one root (b) two roots (c) no roots. Which is correct a, b or c? 14.....

15. A certain series of license numbers is to be formed in the following way. Each license number is to be composed of one letter of the alphabet followed by a 3-digit number not starting with zero. How many different license numbers will result if, in the complete series, all the letters of the alphabet are used and repetition of the digits is allowed? 15.....

16. Find the number of combinations of 40 things taken 38 at a time. 16.....

17. If the four volumes of the *Life of Lincoln* were placed on a library shelf at random, what is the probability that they would be in the order I, II, III, IV? 17.....

18. Write an equation whose roots are the roots of $x^3+8x^2-32 = 0$ each divided by 2. 18.....

19. Write an equation whose roots are the roots of $x^3-6x^2+2x+4 = 0$ each decreased by 2. 19.....

20. How many real roots has the equation $x^4-3x^2-5 = 0$? 20.....

Part II

Answer five questions from part II.

21. Solve the equation $2x^4-x^3+x^2-2x-6 = 0$ [10]

22. Find to the *nearest tenth* the positive root of the equation $x^3+6x^2-4x-8 = 0$. [10]

23. *a.* Draw the graph of $y = 2^x$ from $x = -2$ to $x = 3$. [6]
b. On the same axes used in answer to *a*, draw the graph of $y = -x+4$ [2]
c. From the graphs made in answer to *a* and *b*, estimate to the *nearest tenth* the value of x that satisfies the equation $2^x = -x+4$ [2]

24. Using the formula $Q = Pe^{-nr}$, find Q to the *nearest hundredth* when $P = 760$, $e = 2.718$, $n = 55$ and $r = .14$ [10]

25. *a.* State and prove the Remainder Theorem. [5]
b. The function $x^3+mx^2+nx+24$ is exactly divisible by $x+2$, but, when it is divided by $x-3$, the remainder is 30. Find m and n. [5]

26. Find the integral value of k such that one root of the equation $x^3-5x^2-2x+k = 0$ shall be 5 more than another. [10]

27. Tom can paint a wall with a sprayer in 5 hours less time than it takes him to paint it by hand. One day he started the job with the sprayer but after 1 hour and 20 minutes, the sprayer became clogged and he finished the job by hand in 6 hours. How long would it have taken him to do the whole job with the sprayer? [10]

*28. *a.* Find the modulus of $-8 - 15i$. [2]
b. Find to the *nearest degree* the amplitude (angle) of $-8 - 15i$. [3]
c. Express $2(\cos 150° + i \sin 150°)$ in the form of $a + bi$. [2]
d. Express one of the roots of $x^2 + 25 = 0$ in polar form. [3]

*29. Given the equation $y = -x^2 + 6x - 1$
a. Find the slope of the secant line drawn through the two points of the graph for which $x = -1$ and $x = 2$. [4]
b. Write the equation of the line tangent to the graph of $y = -x^2 + 6x - 1$ at the point for which $x = 1$. [6]

*This question is based upon one of the optional topics in the syllabus.

The University of the State of New York

320TH HIGH SCHOOL EXAMINATION

TWELFTH YEAR MATHEMATICS

Tuesday, January 26, 1954 — 9.15 a. m. to 12.15 p. m., only

Note to teacher: These questions may be used in conjunction with the regular Regents examination in advanced algebra by those pupils who have followed the outline in the twelfth year syllabus. A copy of this sheet should be distributed to each pupil qualified, together with a copy of the regular examination paper in advanced algebra. If sufficient copies of this sheet are not available, these questions may be written on the blackboard.

Part I

Directions: Since questions 18, 19 and 20 on the examination in advanced algebra are not based on topics in the twelfth year syllabus, you may replace one or more of those by any of the following questions. Indicate any substitutions by labeling the answers *A*, *B* or *C*. (Write answers on the regular question paper opposite the questions you are replacing.)

A Find the slope of a line that is perpendicular to the line $2x - 3y = 7$.

B Find the equation of the circle whose center is $(-2, 3)$ and whose radius is 5.

C Express in polar form the result of dividing $8(\cos 30° + i \sin 30°)$ by $2(\cos 10° + i \sin 10°)$.

Part II

Directions: The following question is based on one of the optional topics of the twelfth year syllabus and may be used in place of *any one* of the questions on part II of the examination in advanced algebra.

30 Answer both *a* and *b*:

a Express in determinant form the area of the triangle whose vertices are $(-2, -1)$, $(3, 1)$ and $(1, 5)$. [5]

b By evaluating the determinant found in answer to part *a*, find the area of the triangle. [5]