The University of the State of New York

311TH High School Examination

ADVANCED ALGEBRA

Wednesday, January 24, 1951 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to part II (a) name of school where you have studied, (b) number of weeks and recitations a week in advanced algebra.

The minimum time requirement is four or five recitations a week for half a school year after the completion of intermediate algebra.

Part II

Answer five questions from part II.

21 Solve the equation $2x^4 - 3x^3 - 6x^2 - 8x - 3 = 0$. [10]

22 Find, to the *nearest tenth*, the positive root of the equation $x^3 + 4x^2 + 3x - 14 = 0$. [10]

23 a On the same set of axes, draw the graphs of $(x - 1)^2 + y^2 = 9$ and $y = 1 - x^2$. [4, 4]

     b From the graphs made in answer to a, estimate, to the *nearest tenth*, the values of x and y satisfying both equations. [2]

24 The time of vibration of a pendulum consisting of a slender uniform rod suspended at one end is given by the formula $t = 2\pi \sqrt{\dfrac{2l}{3g}}$, where t is the time in seconds, l is the length in feet and g is a constant. Using logarithms, find, to the *nearest hundredth of a foot*, the length of a pendulum whose time of vibration is 2.25 seconds. [Use $g = 32.16$ and $\pi = 3.14$] [10]

25 In a geometric progression of four terms, the sum of the first and third terms is 65 and the sum of the second and fourth terms is $97\frac{1}{2}$. Find the four terms of the progression. [10]

26 A man found that he could mow his lawn in 3 hours less time with his power mower than with his hand mower. One day his power mower broke down 45 minutes after he started mowing the lawn. It then took him $3\frac{1}{2}$ hours to complete the job with his hand mower. How many hours does it take to mow the lawn with the power mower? [7, 3]

27 A builder has a mixture of 4 parts gravel to 3 parts sand and another mixture of 2 parts gravel to 1 part sand. How many cubic feet of each of these mixtures should he use to form 25 cubic feet of a mixture of 3 parts gravel to 2 parts sand? [7, 3]

*28 a Find the modulus of $12 - 5i$. [2]

    b Find to the *nearest degree* the amplitude (angle) of $12 - 5i$. [3]

    c Express $2(\cos 135° + i \sin 135°)$ in the form $a + bi$. [2]

    d Express one of the imaginary roots of $x^5 - 1 = 0$ in polar form. [3]

*29 Given the graph of $y = ax^2 + bx + c$.

In terms of a and b write an expression for

    (1) the slope of the secant line drawn through the two points of the graph for which $x = 3$ and $x = 5$ [6]

    (2) the slope of the line tangent to this graph at the point for which $x = 3$ [4]

* This question is based upon one of the optional topics in the syllabus.

[2]

Fill in the following lines:

Name of pupil ....Name of school...

Part I

Answer all questions in this part. Each correct answer will receive 2½ credits. No partial credit will be allowed.

1 Write an equation of the line through the point $(-3, 4)$ and parallel to the line $3x + 4y = 6$.

1................

2 If $f(x) = 2x - 3$, find $f(1 - a)$.

2................

3 Find the remainder when $4x^9 - 3x^4 + 2$ is divided by $x + 1$.

3................

4 Express in simplest form the third term in the expansion of

$$\left(\frac{1}{y} + \sqrt{x}\right)^7$$

4................

5 Express the repeating decimal $.232323\ldots$ as a common fraction.

5................

6 If x, y and z are the first three terms of a geometric progression of positive terms, express y in terms of x and z.

6................

7 The third term of an arithmetic progression whose common difference is 3 is one fourth of the tenth term. Find the first term.

7................

8 If y varies inversely as the product of x and z, and $y = 6$ when $x = 2$ and $z = 3$, find y when $x = 3$ and $z = 4$.

8................

9 Find a value of x satisfying the equation $x^{\frac{4}{3}} = 81$.

9................

10 Solve $5^x = 12$ for x to the *nearest tenth*.

10................

11 Transform the equation $2x^3 - 9x^2 - 7 = 0$ into an equation whose roots are each less by 2 than the roots of the given equation.

11................

12 Transform the equation $x^3 - x^2 - x - 1 = 0$ into an equation whose roots are each three times the roots of the given equation.

12................

13 Two roots of the equation $x^3 + px^2 + qx + r = 0$ are 3 and $2 - 4i$: and p, q and r are real numbers. What is the value of p?

13................

14 The complex number $12 - 18i$ is represented graphically by point P. What complex number is represented by the mid-point of the line segment joining the origin and point P?

14................

Directions (questions 15-17) — Indicate the correct answer to *each* question by writing on the line at the right the letter a, b or c.

15 If the product of $2 + i$ and $3 - 2i$ is expressed in the form $p + qi$, the value of p is $(a)6$ $(b)8$ $(c)8 - i$

15......

16 The coefficients p, q and r of the equation $x^6 + px^4 + qx^2 + r = 0$ are integers, two of them being positive and the third negative. If the equation has only one negative root, the negative coefficient must be $(a)p$ $(b)q$ $(c)r$

16......

17 The fraction $\dfrac{\log a}{\log b}$ is equal to $(a)\dfrac{\log 2a}{\log 2b}$ $(b)\dfrac{\log a^2}{\log b^2}$

$(c)\log \dfrac{a}{b}$

17......

18 From a group of 5 men and 4 women, how many different committees can be formed each consisting of 2 men and 2 women?

18................

19 Find the number of integers between 400 and 1000 that have 6 for the second digit.

19................

20 If a date in January is selected at random, what is the probability that the number representing the date is exactly divisible by 10?

20................