ADVANCED ALGEBRA

Wednesday, January 28, 1948 — 9.15 a. m. to 12.15 p. m., only

Part I

Answer all questions on this part. Each correct answer will receive $2\frac{1}{2}$ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1. Write the equation of the straight line passing through the point (4, 3) and parallel to the line $2y = 5x - 2$ 1______

2. In how many points does the graph of $y = x^2 + 3$ intersect the graph of $xy = 12$? 2______

3. Find, to the *nearest hundredth*, the value of $\sqrt[3]{.456}$ 3______

4. Solve for x the equation $3^x = \frac{1}{9}$ 4______

5. Express $\dfrac{3 + 2i}{2 - i}$ in the form $a + bi$. 5______

6. The complex numbers $2 + 2i$ and $1 + i\sqrt{3}$ are represented graphically by points A and B. If lines are drawn from A and B to the origin O, find the number of degrees in angle AOB. 6______

7. Write in simplest form the *fifth* term of $(x + \dfrac{2}{x})^8$ 7______

8. Find the remainder when $x^{26} - 5$ is divided by $x + 1$ 8______

9. Write the equation of lowest possible degree with rational coefficients two of whose roots are 2 and $\sqrt{5}$ 9______

10. Transform the equation $x^3 - 5x^2 - 7x + 2 = 0$ into an equation whose roots are the roots of the given equation each multiplied by 3. 10______

11. Transform the equation $x^3 + 5x^2 - 20x - 10 = 0$ into an equation whose roots are those of the original equation each diminished by 1. 11______

12. How many negative roots has the equation $x^3 - 3x^2 + 2 = 0$? 12______

13. Two of the roots of the equation $x^3 - px + q = 0$ are 4 and 3. Find the third root. 13______

14. The period of a pendulum varies directly as the square root of its length. If the period is 2 seconds when the length is 81 centimeters, find the number of seconds in the period when the length is 25 centimeters. 14______

15. Find the value of $2x^0 + (\dfrac{9}{x})^{-2} + \sqrt[3]{x^2}$ when $x = 27$ 15______

16. If $f(x) = 3x^3 - 2x + 5$, find $f(2)$. 16______

17. The third term of a geometric series is 12 and the sixth term is -96. Find the *fourth* term. 17______

18. How many different committees of 4 men each can be selected from a group of 12 men? 18______

19. How many different numbers each ending in 3 and greater than 1000 can be written with the digits 1, 2, 3 and 4 if no digit is repeated in any number? 19______

20. In a group of children there are 4 boys and 3 girls. If one child is selected by lot, what is the probability that a girl will be selected? 20______

Part II

Answer five questions from part II.

21. Find, to the *nearest tenth,* the positive root of $2x^3 - 6x - 1 = 0$ [10]

22. Solve completely the equation $x^4 + x^3 - x^2 - 7x - 6 = 0$ [10]

23. The number n of vibrations per second made by a stretched string is given by the relation $n = \frac{1}{2L}\sqrt{\frac{Mg}{m}}$ where L is the length of the string, M the weight used to stretch the string, m the weight of one centimeter of the string and $g = 980$. Using logarithms, find n to the *nearest integer,* when $M = 7285$ grams, $L = 76.8$ centimeters and $m = 0.0059$ gram. [10]

24. *a.* Using the same set of axes, draw the graphs of $(x + 1)^2 + y^2 = 9$ and $x^2 - y^2 = 4$ [4, 4]

b. From the graphs made in answer to *a,* find the negative value of x and the negative value of y that satisfy both equations. [Express any approximate value to the *nearest tenth.*] [2]

25. A number of young men purchased a camp, each paying the same amount. If there had been two more men in the company, each would have paid $12 less. If there had been three fewer men in the company, each would have paid $24 more. How many men were there and how much did each pay? [10]

26. Three men, A, B and C, set out from the same place and traveled over the same route at the rates of 4, 5 and 6 miles per hour, respectively. B started 2 hours later than A. How long after B started must C have set out if B and C overtook A at the same moment? [10]

27. Prove that if $a + bi$ is a root of $x^3 + px + q = 0$, in which p and q are real numbers, then $a - bi$ is also a root. [The statement that this is a special case of a more general theorem will not be accepted as proof.] [10]

*28. *a.* Express $2(\cos 150° + i \sin 150°)$ in the form $a + bi$. [4]

b. Express $1 + i\sqrt{3}$ in polar form. [3]

c. Add graphically $-6 + 2i$ and $3 + 3i$. [3]

*29. An open box is to be made from a sheet of tin 12 inches square by cutting equal squares from the four corners and bending up the sides.

a. If x represents a side of the square to be cut out and if y represents the volume of the box, express y as a function of x. [2]

b. Find the derivative of y with respect to x. [4]

c. Find the value of x for which the volume of the box will be a maximum. [4]

* This question is based on one of the optional topics in the syllabus.