ADVANCED ALGEBRA

Wednesday, January 30, 1946 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to part II (a) name of school where you have studied, (b) number of weeks and recitations a week in advanced algebra.

The minimum time requirement is five recitations a week for half a school year after the completion of intermediate algebra.

Part II

Answer five questions from part II.

21 Find, correct to the *nearest tenth*, the root of the equation $x^3 + 3x^2 + 4x + 3 = 0$ that lies between —1 and —2. [10]

22 Solve the equation $x^4 - 6x^3 + 17x^2 - 28x + 20 = 0$ [10]

23 a State and prove the Remainder Theorem. [6]
    b Determine k so that when $x^3 - 3x^2 + 5x - k$ is divided by $x - 2$, the remainder is 3. [2]
    c If $x - 2$ exactly divides $x^4 + kx^3 - 3x^2 - 4x + 12$, what is the value of k? [2]

24 Solve the equation $6^x = (4.7)^{2x + 3}$. Express the root correct to the *nearest tenth*. [10]

25 The area of a triangle is 36 sq. in. If the base is increased by 1 inch and the altitude is decreased by 2 inches, the area becomes 30 sq. in. Find the base and the altitude of the original triangle. [10]

26 a Express, in terms of k, the values of x common to $y = x^2 - 6x + 5$ and $y = 2x + k$ [4]
    b Draw the graph of the equation $y = x^2 - 6x + 5$ [3]
    c From the result obtained in answer to a, choose the value of k so that the graph of $y = 2x + k$ will be tangent to the graph of $y = x^2 - 6x + 5$ [2]
    d Using the same set of axes as in b, draw the graph of the equation obtained in answer to c. [1]

[1]

27 a State whether *each* of the following statements is *true* or *false*:
  (1) The expression "*y* varies directly as *x*," when represented graphically, is a straight line. [2]
  (2) The expression "*y* varies inversely as *x*," when represented graphically, is a hyperbola. [2]

  b The force of attraction F between two bodies varies directly as the product of their masses m and M and inversely as the square of the distance d between them. If $F = 6$ when $m = 64$, $M = 108$ and $d = 24$, find m if $F = 96$, $M = 256$ and $d = 18$ [6]

*28 a Express the roots of $x^3 = 8$ in polar form. [4]
  b Represent graphically the roots obtained in answer to *a*. [3]
  c Express $(2,240°)$ in the form $a + bi$. [3]

*29 A telephone company finds that there is a net profit of $15.50 per instrument if an exchange has 900 or fewer subscribers. If there are above 900 subscribers, the net profit per instrument decreases one cent for each subscriber above that number.
  a If x represents the number of subscribers above 900, express the net profit y in terms of x. [5]
  b For the expression obtained in answer to *a*, find $\dfrac{dy}{dx}$ [2]
  c Find the total number of subscribers for which the net profit is a maximum. [3]

* This question is based on one of the optional topics in the syllabus.

Fill in the following lines:

Name of school......................................Name of pupil...

Part I

Answer all questions in this part. Each correct answer will receive 2½ credits. **No partial credit will** be allowed. Each answer must be reduced to its simplest form.

1 Find the x intercept of the line whose equation is $6y - x - 10 = 0$

1.....................

2 Find the equation of the line through the point $(-3, 4)$ and parallel to the line whose equation is $3y = x + 7$

2.....................

3 Find, correct to the *nearest hundredth*, the value of $\sqrt[3]{39.7}$

3.....................

4 Express $\log \dfrac{a^2 b}{\sqrt[3]{c}}$ in terms of the logarithms of a, b and c.

4.....................

5 Express $\dfrac{6 + 5i}{1 - 2i}$ as a fraction with a real denominator.

5.....................

6 Write the equation of lowest degree possible with real coefficients which has for three of its roots 2, -2 and $3i$.

6.....................

7 How many imaginary roots has the following equation?
$x^6 + 2x^4 + 3x^2 + 4 = 0$

7.....................

8 What is the remainder when $3.4x^{25} + 8.5$ is divided by $x + 1$?

8.....................

9 Between what two consecutive integers does the positive root of $x^3 + 5x^2 - 4x - 5 = 0$ lie?

9.....................

10 Find the negative root of the equation $x^3 - 7x^2 + 36 = 0$

10.....................

11 If $f(x) = x^4 + 3x^2 + 10$, find $f(-x)$.

11.....................

12 Transform the equation $x^3 + 3x^2 - 4x + 1 = 0$ into an equation whose roots are greater by 1 than those of the given equation.

12.....................

13 Transform the equation $x^3 - 2x^2 - 5x + 6 = 0$ into an equation whose roots are the roots of the given equation each multiplied by 2

13.....................

14 How many odd numbers of three digits each can be formed from 1, 2, 4, 6 if no repetition of digits in any one number is allowed?

14.....................

15 Find the *sixth* term of the expansion of $\left(\dfrac{a}{b} + \dfrac{b}{a} \right)^{10}$

15.....................

16 From 14 men, how many different groups of 11 men can be chosen?

16.....................

17 If an event can occur in a different ways and fail to occur in b different ways and all $(a + b)$ ways are equally likely, what is the probability that the event will occur at a given trial?

17.....................

18 What is the common difference in an arithmetic progression whose first term is 4 and whose ninth term is 28?

18.....................

19 In how many points does the graph of $x^2 + 4y^2 = 16$ intersect the graph of $y^2 = 4x$?

19.....................

20 Find the positive value of x which satisfies the equation $2^{x^2 - x - 4} = 4$

20.....................