The University of the State of New York

290TH HIGH SCHOOL EXAMINATION

ADVANCED ALGEBRA

Wednesday, January 19, 1944 — 9.15 a. m. to 12.15 p. m., only

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Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to part II (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in advanced algebra.

The minimum time requirement is five recitations a week for half a school year after the completion of intermediate algebra.

Part II

Answer five questions from part II.

21 Find, correct to the *nearest tenth*, the positive root of the equation
$x^3 + x^2 - 2x - 11 = 0$ [10]

22 The roots of the equation $x^3 - 9x^2 + 2x + k = 0$ are in arithmetic progression. Find the value of k. [10]

23 Find the four roots of the equation $x^4 - 5x^3 + 9x^2 + x - 14 = 0$ [10]

24 *a* Find, correct to the *nearest tenth*, the value of $\sqrt[4]{\frac{2.473 \times 5.86^{2.1}}{.007968}}$ [6]

b Find the value of x correct to the *nearest tenth*: $3.4^x = .16$ [4]

25 Prove that if $a + bi$ is a root of the equation $px^3 + qx^2 + rx + s = 0$, in which p, q, r and s are real numbers, then $a - bi$ is also a root. [The statement that this is a special case of the more general theorem will not be accepted as a proof.] [10]

26 A man inherits $15,000. A percentage of this amount is deducted as an inheritance tax and a percentage of the remainder for legal fees. The percentage deducted for fees is 2% higher than that for the tax. If the amount finally received is $11,880, find the amount of the inheritance tax. [10]

27 *a* Using the same set of axes, draw the graphs of $(x-3)^2 + y^2 = 9$ and $xy = 8$ [4, 4]

b From the graphs made in answer to *a*, find, correct to the *nearest tenth*, the real values of x and y that satisfy both equations. [2]

*28 Given $f(x) = x^3 + x - 7$

a Find the first and the second derivatives of $f(x)$. [2, 1]

b Find the slope of the line that is tangent to the graph of $f(x)$ at the point whose abscissa is 1. [2]

c Find the rate of change of $f(x)$ when $x = -1$ [2]

d Does $f(x)$ have a maximum and a minimum value? [1]

e At what point is the slope of the line which is tangent to the graph of $f(x)$ a minimum? [2]

*29 *a* Express $-1 + i\sqrt{3}$ in the form $\rho(\cos\theta + i\sin\theta)$. [2]

b Express $4(\cos 240° + i \sin 240°)$ in the form $a + bi$. [2]

c Perform the indicated operations:

$$\frac{[4(\cos 240° + i \sin 240°)]^2}{-1 + i\sqrt{3}}$$ [6]

* This question is based on one of the optional topics in the syllabus.

Fill in the following lines:

Name of school.....Name of pupil.....

Part I

Answer all questions in this part. Each correct answer will receive 2½ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1 Express $\frac{1+i}{2-3i}$ in the form $a+bi$. 1.....

2 If $f(x) = x^3 + 2$, find $f(2a-1)$. 2.....

3 Find the remainder when $2x^7 + 2x^3 - 3$ is divided by $x - 1$. 3.....

4 For what values of k are the roots of the equation $2x^2 + 3kx = -2$ equal? 4.....

5 Is $x = 1$ a root of the equation

$$(\frac{x}{x-1})^2 - 5(\frac{x}{x-1}) + 4 = 0?$$ 5.....

6 Solve the formula $E = \frac{1}{t}(\frac{T^2}{R} - 1)$ for the positive value of T. 6.....

7 Find the rational root of the equation $x^3 - 7x^2 + 8x + 4 = 0$ 7.....

8 Write the equation whose roots are less by 2 than the roots of the equation $2x^3 - 3x^2 - 10x - 3 = 0$ 8.....

9 Write the equation whose roots are the negatives of the roots of the equation $x^4 + x^3 - 10 = 0$ 9.....

10 Express $\log(a^2 - b^2)$ as the sum of two logarithms. 10.....

11 Find, correct to the *nearest tenth*, $\log_2 3$. 11.....

12 Write the equation of the straight line whose x intercept is 3 and whose y intercept is -3. 12.....

13 Find the coordinates of the mid-point of the line segment the coordinates of whose end points are $(-2, 7)$ and $(2, 3)$. 13.....

14 Find the *third* term in the expansion of $(\sqrt{x} + \frac{2}{y})^5$ 14.....

15 A basketball squad consists of 10 men. In how many ways can a team of 5 men be selected from the squad if only one of the men can play center? 15.....

16 Four men, A, B, C and D, stand in a row. What is the probability that A and B are next to each other? 16.....

17 If y varies directly as x and inversely as z and if $y = 8$ when $x = 2$ and $z = 3$, find y when $x = 5$ and $z = 6$ 17.....

Directions (questions 18–20) — If the blank in each statement is replaced by one of the words *always*, *sometimes* or *never*, the resulting statement will be true. Select the word that will correctly complete *each* statement and write this word on the line at the right.

18 The equation $f(x) = 0$ ... has as many negative roots as there are variations of sign in $f(-x)$. 18.....

19 If any three numbers are in geometric progression, their reciprocals, taken in the same order, are ... in geometric progression. 19.....

20 The graphs of $y = ax^2$ and $y = b$, when drawn on the same set of axes, will ... intersect if a is positive and b is negative. 20.....