The University of the State of New York

286th High School Examination

ADVANCED ALGEBRA

Wednesday, January 20, 1943 — 9.15 a. m. to 12.15 p. m., only

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Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to part II (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in advanced algebra.

The minimum time requirement is five recitations a week for half a school year after the completion of intermediate algebra.

Part II

Answer five questions from part II.

21 One root of the equation $x^4 - 2x^3 - x^2 - 2x - 2 = 0$ is i. Find the other three roots. [10]

22 Find, correct to the *nearest tenth,* the real root of the equation $x^3 - 6x - 12 = 0$ [10]

23 *a* On the same set of axes draw the graphs of the following equations: $x^2 + y^2 = \frac{169}{4}$ and $4y^2 = 4x + 1$ [1, 4]

b From the graphs made in answer to *a*, find the real values of x and y common to the two equations. [2]

c Check the results obtained in answer to *b* by solving the equations algebraically. [3]

24 Given the formula $V = 127\, r^{0.12}\sqrt{rs}.$ If $r = 4.56$ and $s = .0693$, find V correct to the *nearest integer.* [10]

25 An airplane has just enough gasoline to travel from its base B to a point P and return. The distance r from B to P is known as the Radius of Action.

a If the speed of the plane on its outward trip is v_1 miles per hour, the speed returning over the same course v_2 miles per hour and the total time of the round trip is t hours, derive a formula for r in terms of v_1, v_2 and t. [7]

b Find, correct to the *nearest mile,* the radius of action of a plane if $v_1 = 150$ m. p. h., $v_2 = 125$ m. p. h. and $t = 3$ hours 20 minutes. [3]

26 When flying at a speed of 170 m. p. h., an airplane consumes 16 gallons of gasoline per hour. If the speed is reduced to 150 m. p. h., the gasoline consumption is reduced to 12 gallons per hour. Using these data, solve the following problem graphically:

An airplane has 120 gallons of gasoline in its tanks. It flies for $5\frac{1}{2}$ hours at 170 m. p. h. and then reduces its speed to 150 m. p. h. At that speed, how much longer can it remain in the air? [10]

27 An antifreeze solution contains 15% crude glycerin and 85% water. How many pints of crude glycerin must be added to two gallons of the solution so that it shall be only 75% water? [Compute the result to the *nearest pint.*] [7, 3]

*28 *a* Write the complex number $6 + 8i$ in polar form, with its amplitude correct to the *nearest degree.* [6]

b Express $2(\cos 60° + i \sin 60°)$ in the form $a + bi$. [4]

*29 If a gun is fired at an angle of 45° to the horizontal and with a muzzle velocity of 1600 feet per second, the path of the projectile is given by the equation $y = x - \frac{x^2}{80{,}000}$ where y represents the height in feet of the projectile above the ground and x represents the horizontal distance in feet traveled by the projectile.

a Determine the maximum height attained by the projectile. [7]

b Find, correct to the *nearest mile*, the distance from the gun to the point where the projectile strikes the ground. [3]

* This question is based on one of the optional topics in the syllabus.

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Fill in the following lines:

Name of school...Name of pupil...

Part I

Answer all questions in this part. Each correct answer will receive $2\frac{1}{2}$ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1 Write the expansion of $(1 + i)^2$ in terms of i. 1...............

2 In how many points does the graph of $x + y = 9$ intersect the graph of $xy = 9$? 2...............

3 Given $f(x) = 3x^{\frac{1}{2}} - (2x)^0 + 16x^{-2}$; write the value of $f(4)$. 3...............

4 If r varies inversely as t and $r = 8$ when $t = 3$, find the value of r when $t = 6$. 4...............

5 Given x, y and z, the first three terms of an arithmetic progression; express y in terms of x and z. 5...............

6 Write the first *three* terms in the expansion of $(x + a)^8$. 6...............

7 Write the equation of the straight line whose y intercept is 3 and which has the same slope as the line whose equation is $y = 2x + 7$. 7...............

8 What is the abscissa of the point at which the graph of the equation $y = \log x$ intersects the x axis? 8...............

9 Find, correct to the *nearest thousandth*, the value of $\sqrt[3]{.7438}$. 9...............

10 For what value of k, other than zero, are the roots of the equation $x^2 + kx + k = 0$ real and equal? 10...............

11 The equation $x^3 - 6x^2 + 3x - 1 = 0$ is transformed into an equation in which each root has been diminished by a. What must be the value of a in order that the sum of the roots of the transformed equation shall be zero? 11...............

12 Transform $8x^3 + 4x^2 + 1 = 0$ into an equation whose roots are twice the roots of the given equation. 12...............

13 For what value of k is the expression $2x^3 + 3x^2 - 7x + k$ exactly divisible by $x - 2$? 13...............

14 If two of the digits 1 to 9, inclusive, are chosen at random, what is the probability that the sum of the two digits will be 10? 14...............

15 How many different signals can be made using two blue flags and three white flags if all five are arranged in a vertical line? 15...............

16 Two fixed posts are d feet apart. If n posts are to be set, equally spaced, between these two, how many feet apart will any two consecutive posts be? 16...............

17 A box with square base is made of sheet metal. A side of the base is x and the volume of the box is V. Express the height h of the box as a function of x and V. [Neglect the thickness of the metal.] 17...............

18 If $s = r^2 - 4$, express $\log s$ in terms of $\log (r + 2)$ and $\log (r - 2)$. 18...............

19 Is the following statement *true* or *false?*
The equation $x^3 + px + q = 0$ has one negative root and two imaginary roots if both p and q are positive and real. 19...............

20 Is the value of a repeating decimal a rational number? [Answer *yes* or *no*.] 20...............