The University of the State of New York

283d High School Examination

ADVANCED ALGEBRA

Wednesday, January 21, 1942 — 9.15 a. m. to 12.15 p. m., only

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Instructions

Do not open this sheet until the signal is given.

Part I

This part is to be done first and the maximum time allowed for it is one and one half hours. Merely write the answer to each question in the space at the right; no work need be shown.

If you finish part I before the signal to stop is given you may begin part II. However, it is advisable to look your work over carefully before proceeding, since *no credit will be given any answer in part I which is not correct and in its simplest form.*

When the signal to stop is given at the close of the one and one half hour period, work on part I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

Part II

Write at top of first page of answer paper to part II (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in advanced algebra.

The minimum time requirement is five recitations a week for half a school year after the completion of intermediate algebra.

The use of the slide rule will be allowed for checking but all computations with tables must be shown on the answer paper.

See instructions for part II on page 1.

Part II

Answer five questions from this part. Full credit will not be granted unless all operations (except mental ones) necessary to find results are given; simply indicating the operations is not sufficient. Each answer should be reduced to its simplest form. Purely arithmetical solutions for problems will not be accepted.

21 Solve the equation $6x^4 - 5x^3 + 7x^2 - 5x + 1 = 0$ [10]

22 Find, correct to the *nearest tenth*, the real root of $x^3 - 3x^2 + 5x - 9 = 0$ [10]

23 Given the set of equations $x^2 + y^2 = 16$

$$x + y = k$$

For what positive value of k will the graph of $x + y = k$ be tangent to the graph of $x^2 + y^2 = 16$? [10]

24 In an infinite series of equilateral triangles, the vertices of each triangle after the first are the midpoints of the sides of the preceding triangle. The perimeter of the first triangle is 3 feet. Find the sum of the perimeters of all the triangles. [10]

25 An automobile uses 9 gallons of gasoline in traveling 150 miles at 25 miles an hour. If gasoline consumption varies jointly as the distance traveled and the square root of the speed, find how many gallons would be used on a like trip of 184 miles at a speed of 36 miles an hour. [10]

26 Given the formula $A = P(1 + r)^n$

a Obtain the value of $\log (1 + r)$ in terms of A, P and n. [2]

b Solve for r when $A = \$1850$, $P = \$1000$ and $n = 8$. [8]

27 Prove that an equation whose coefficients are integers, that of the term of highest degree being unity, can not have as a root a rational fraction in its lowest terms. [10]

*28 *a* Write the complex number $3 + i$ in polar form, with amplitude correct to the *nearest degree*. [6]

b Write the number $2(\cos 30^\circ + i \sin 30^\circ)$ in the form $a + bi$. [4]

*29 If an arrow is shot vertically upwards with an initial velocity of 144 feet per second, the height S reached in t seconds can be found from the formula $S = 144t - 16t^2$. Using this formula, find (*a*) the velocity of the arrow after 3 seconds, (*b*) the height to which it will rise. [5, 5]

*This question is based on one of the optional topics in the syllabus.

Fill in the following lines:

Name of school.....Name of pupil.....

Detach this sheet and hand it in at the close of the one and one half hour period.

Part I

Answer all questions in this part. Each correct answer will receive 2½ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1 For what value of a is the graph of $ax + 2y + 3 = 0$ parallel to the x axis? 1.....

2 Express the product of two consecutive integers as a function of n in which n is the smaller integer. 2.....

3 For what values of x is $(x - 5)(x - 2)$ equal to $x - 5$? 3.....

4 Write the value of $y^0 (x^{-1})^{-1}$ when $y = 8$ and $x = 27$. 4.....

5 Write the product $(1 + i)(3 + 2i)$ in the form $a + bi$. 5.....

6 The fifth term of an arithmetic progression is 9 and the tenth term is 29; write the *sixth* term. 6.....

7 For what value of k is $x^3 + 3x^2 + kx - 5$ divisible by $x - 1$? 7.....

8 Find the *sum* of the first two terms of the expansion $(1 + \frac{1}{x})^x$. 8.....

9 In how many ways can 3 questions be chosen out of 5 questions? 9.....

10 How many radio stations can be named if the 26 letters of the alphabet are used 3 different letters at a time? 10.....

11 There are four equally convenient roads from A to B. If two automobiles make the trip, what is the probability that both take the same road? 11.....

12 Solve for x the equation $3^{x-1} = 243$ 12.....

13 Given $\log x = a$ and $\log y = b$; express $\log \frac{x^2}{y}$ in terms of a and b. 13.....

14 Is the following a correct statement of Descartes's rule of signs: "An equation has as many positive roots as it has variations in signs"? [Answer *yes* or *no*.] 14.....

15 If the roots of the equation $x^3 + px^2 + qx + r = 0$ are a, b and c, express p in terms of a, b and c. 15.....

16 Write the equation whose roots are greater by 2 than the roots of $x^3 + 4 = 0$. 16.....

17 Write the equation whose roots are twice the roots of $4x^3 - 3x + 1 = 0$. 17.....

Questions 18–20 refer to the equation $f(x) = 10x^5 + cx^3 + ex + 21 = 0$, where c and e are positive integers. [Answer *yes* or *no*.]

18 Is -3 a possible root of $f(x) = 0$? 18.....

19 Do the values of c and e affect the sum of the roots of $f(x) = 0$? 19.....

20 Must the real root (or roots) of $f(x) = 0$ be of fractional form with denominator a factor of 10? 20.....