# ADVANCED ALGEBRA

Wednesday, January 24, 1940 — 9.15 a. m. to 12.15 p. m., only

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## Instructions

*Do not open this sheet until the signal is given.*

### Group I

*This group is to be done first and the maximum time allowed for it is one and one half hours.* Merely write the answer to each question in the space at the right; no work need be shown.

If you finish group I before the signal to stop is given you may begin group II. However, it is advisable to look your work over carefully before proceeding, since *no credit will be given any answer in group I which is not correct and in its simplest form.*

When the signal to stop is given at the close of the one and one half hour period, work on group I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

### Group II

Write at top of first page of answer paper to group II (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in advanced algebra.

The minimum time requirement is five recitations a week for half a school year after the completion of intermediate algebra.

The use of the slide rule will be allowed for checking but all computations with tables must be shown on the answer paper.

### Fill in the following lines:

Name of school............................................................Name of pupil..........................................................

**Detach this sheet and hand it in at the close of the one and one half hour period.**

### Group I

*Answer all questions in this group. Each correct answer will receive 2½ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.*

1 What is the $y$ intercept of the line whose equation is $2x - 3y = 6$?  1.....................

2 What is the slope of the line whose equation is $\dfrac{x}{3} - \dfrac{y}{2} = 1$?  2.....................

3 What is the equation of the line which passes through the point $(0, -2)$ and whose slope is $\frac{2}{3}$?  3.....................

4 What is the arithmetic mean between $2a$ and $2b$?  4.....................

5 Insert a positive geometric mean between 1.28 and 128.  5.....................

6 What is the remainder when $2x^{33} + 35$ is divided by $x + 1$?  6.....................

7 Write the first *three* terms of the expansion $(1 + x)^n$  7.....................

8 What is the real value of $\left(\dfrac{2^0}{8^{\frac{1}{3}}}\right)^{-1}$?  8.....................

9 According to Descartes' rule of signs, *must* the equation $x^4 - 3x^3 + 5x^2 - 7x + 16 = 0$ have four positive roots? [Answer *Yes* or *No*.]  9.....................

10 Transform the equation $x^4 + 8x^3 + 23x^2 + 28x + 13 = 0$ into an equation whose roots are greater by 2 than the roots of the given equation.  10.....................

11 If $p$ and $-q$ are the roots of $ax^2 + bx + c = 0$, what are the roots of the equation $ax^2 - bx + c = 0$?  11.....................

12 Write the equation whose roots are one half the roots of the equation $x^3 - 8x + 8 = 0$  12.....................

13 Two roots of an equation of the fourth degree with real coefficients are $2 + i$ and $-2 + i$. What are the other roots?  13.....................

14 If $x = \log 3$, what is the value of $10^x$?  14.....................

15 Find, correct to the *nearest tenth*, the value of $\sqrt[3]{5.402}$  15.....................

16 In how many points does the graph of $x^2 - y^2 = 16$ intersect the graph of $xy = 16$?  16.....................

17 Write as an equation the following statement: The weight $(W)$ of an object varies inversely as the square of its distance $(d)$ from the center of the earth.  17.....................

18 On an algebra examination a student is allowed to choose 5 questions out of 9. In how many ways can he choose the 5 questions?  18.....................

19 Five discs in a bag are numbered 1 to 5. What is the probability that the sum of the numbers on three discs picked at random will be greater than 10?  19.....................

20 In an equation of the third degree only the first two terms are legible: $x^3 - 3x^2 \ldots = 0$
Two roots of the equation are known to be 2 and $-3$. What is the third root?  20.....................

[3]

### See instructions for group II on page 1.

### Group II

*Answer five questions from this group. Full credit will not be granted unless all operations (except mental ones) necessary to find results are given; simply indicating the operations is not sufficient. Each answer should be reduced to its simplest form. Purely arithmetical solutions for problems will not be accepted.*

21 Solve completely: $3x^4 + 8x^3 - 9x^2 - 16x + 6 = 0$    [10]

22 Find, correct to the *nearest tenth*, the real root of the equation $2x^3 - 6x - 9 = 0$    [10]

23 Laplace's formula for determining the velocity of sound in air (in meters per second) is $V = \sqrt{\dfrac{kP}{d}}$, where $P$ is the barometric pressure in dynes per square centimeter and $d$ is the density of the air in grams per cubic centimeter. If $P = 1,013,000$, $d = .0013$ and $k = .000142$, by the use of logarithms find $V$ correct to the *nearest integer*.    [10]

24 Given: $(c + id)^3 + q(c + id) + r = 0$, in which $q$ and $r$ are real
   Prove: $(c - id)^3 + q(c - id) + r = 0$    [10]

25 A and B started at the same time to walk from two towns 12 miles apart. They walked in the same direction along the same road and A overtook B six hours after they started. Had they walked toward each other they would have met in two hours. What were their rates of walking?    [10]

26 Given:    $x + y + z = 100$
   $10x + 3y + \frac{1}{2}z = 100$
   a Express $x$ as a function of $y$.    [6]
   b If $x$, $y$ and $z$ are positive integers, determine the values of $x$, $y$ and $z$ that will satisfy the given equations.    [4]

27 a On the same set of axes, plot the graphs of $xy = 16$ and $y^2 = 6x$    [4, 4]
   b From the graphs made in answer to *a*, estimate, correct to the *nearest tenth*, the values of $x$ and $y$ common to the two equations.    [2]

*28 a Find the modulus of $5 + 12i$    [2]
   b Express $1 - i$ in polar form.    [4]
   c Express $2(\cos 90° + i \sin 90°)$ in the form $a + bi$.    [4]

*29 On an examination in advanced algebra the grades (to the nearest 5%) earned by 100 pupils were distributed as follows:

| Grades | 55 | 60 | 65 | 70 | 75 | 80 | 85 | 90 | 95 | 100 |
|---|---|---|---|---|---|---|---|---|---|---|
| Number of pupils | 2 | 3 | 5 | 12 | 16 | 24 | 14 | 10 | 8 | 6 |

   a Which grade most nearly represents the mode?    [2]
   b Which grade most nearly represents the median?    [2]
   c Compute the arithmetic mean.    [5]
   d Is the distribution fairly "normal"?    [1]

\* This question is based on one of the optional topics in the syllabus.