The University of the State of New York

274TH HIGH SCHOOL EXAMINATION

ADVANCED ALGEBRA

Monday, January 23, 1939 — 9.15 a. m. to 12.15 p. m., only

Instructions

Do not open this sheet until the signal is given.

Group I

This group is to be done first and the maximum time allowed for it is one and one half hours. Merely write the answer to each question in the space at the right; no work need be shown.

If you finish group I before the signal to stop is given you may begin group II. However, it is advisable to look your work over carefully before proceeding, since *no credit will be given any answer in group I which is not correct and in its simplest form.*

When the signal to stop is given at the close of the one and one half hour period, work on group I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

Group II

Write at top of first page of answer paper to group II (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in advanced algebra.

The minimum time requirement is five recitations a week for half a school year after the completion of intermediate algebra.

The use of the slide rule will be allowed for checking but all computations with tables must be shown on the answer paper.

Fill in the following lines:

Name of school..... Name of pupil.....

Detach this sheet and hand it in at the close of the one and one half hour period.

Group I

Answer all questions in this group. Each correct answer will receive 2½ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1 What is the slope of the line $x - 2y = 3$? 1.....

2 What is the equation of the line passing through the point $(2, -3)$ and parallel to the line $y = 2x - 3$? 2.....

3 By how much does the arithmetic mean between 16 and 4 exceed the positive geometric mean? 3.....

4 If $f(x) = x^3 - x + 3$, what is the value of $f(-1)$? 4.....

5 Find the value of $64^{\frac{1}{3}} + 5^0 - (\frac{1}{2})^{-1}$. 5.....

6 Write the *third* term of the expansion $(x + \frac{1}{x})^8$ 6.....

7 What is the sum of the roots of the equation
$2x^4 - 4x^3 + 3x^2 - 7 = 0$? 7.....

8 Write the equation whose roots are twice the roots of the equation
$x^3 - \frac{x^2}{2} + \frac{3x}{4} - \frac{1}{4} = 0$ 8.....

9 Write the equation whose roots are greater by 3 than the roots of the equation $x^3 + 3x^2 + 9x + 26 = 0$ 9.....

Directions (questions 10–14) — Answer *yes* or *no*.

10 Is the product of two complex numbers always a real number? 10.....

11 Is the number $-\sqrt{2}$ real? 11.....

12 Is the number 0.272727 ... rational? 12.....

13 Does the equation $x^3 - 3x^2 + 7 = 0$ have any rational fractional roots? 13.....

14 Does the graph of $y^2 = -4(x - 1)$ intersect the graph of $y = 2$? 14.....

15 Write the equation of the lowest possible degree with real coefficients which has for two of its roots 1 and $1 + i$ 15.....

16 Find, correct to the *nearest hundredth*, the value of $\sqrt[3]{.0436}$ 16.....

17 How many odd numbers of three digits each can be formed from the digits 1, 2, 3, 4 if repetitions are not allowed? 17.....

18 Write as an equation the following statement: The area A of a circle varies directly as the square of its radius r. 18.....

19 In a bus accident three out of the 20 passengers were reported injured. A family of three was in the bus. What is the probability that the three persons reported injured belonged to that family? 19.....

20 If $x = \log_e a$, $y = \log_e b$ and $z = \log_b a$, write z as a function of x and y. 20.....

See instructions for group II on page 1.

Group II

Answer five questions from this group. Full credit will not be granted unless all operations (except mental ones) necessary to find results are given; simply indicating the operations is not sufficient. Each answer should be reduced to its simplest form. Purely arithmetical solutions for problems will not be accepted.

21 Find, correct to the *nearest tenth*, the real root of the equation $x^3 - 3x - 14 = 0$ [10]

22 Solve the equation $6x^4 + x^3 - 37x^2 - 6x + 6 = 0$ [10]

23 *a* On the same set of axes, plot the graphs of $y = x^3 - x^2 - 6x + 6$ and $3y = x - 1$ [7, 1]

b From the graphs made in answer to *a*, determine the values of x and y common to the two equations. [Estimate the irrational values correct to the *nearest tenth.*] [2]

24 If in the equation $x^3 + px + q = 0$, p and q are integers, prove that the equation can not have rational fractional roots. [The statement that this is a special case of the more general theorem will not be accepted as proof.] [10]

25 If the rate of income r on an investment I is reduced by d, show that the amount a by which the investment must be increased to realize the same return is given by $a = \frac{dI}{r - d}$ [10]

26 The breaking load P in tons which will crush a cast-iron strut with round ends is given by the formula $P = \frac{50\, d^{3.6}}{L^{1.7}}$ where d is the number of inches in the diameter and L is the number of feet in the length. Find P correct to the *nearest ton* when $d = 4$ and $L = 20$ [10]

27 In a 220-yard dash, A gives B a handicap of 10 yards and beats him by 4 seconds. Had A given B a handicap of 2 seconds, he would have beaten him by 26 yards. Find B's rate in yards per second. [10]

*28 Express the roots of the equation $x^3 - 8 = 0$ in polar form. [10]

*29 Given $y = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x + 2$

a Find the coordinates of the maximum point. [3]

b Find the coordinates of the minimum point. [3]

c Find the coordinates of the point of inflection. [2]

d Sketch the curve. [2]

* This question is based on one of the optional topics in the syllabus.