

The University of the State of New York

271ST HIGH SCHOOL EXAMINATION

**ADVANCED ALGEBRA**

Wednesday, January 26, 1938 — 9.15 a. m. to 12.15 p. m., only

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**Instructions**

*Do not open this sheet until the signal is given.*

**Group I**

*This group is to be done first and the maximum time allowed for it is one and one half hours.*

Merely write the answer to each question in the space at the right; no work need be shown.

If you finish group I before the signal to stop is given you may begin group II. However, it is advisable to look your work over carefully before proceeding, since *no credit will be given any answer in group I which is not correct and in its simplest form.*

When the signal to stop is given at the close of the one and one half hour period, work on group I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

**Group II**

Write at top of first page of answer paper to group II (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in advanced algebra.

The minimum time requirement is five recitations a week for half a school year after the completion of intermediate algebra.

The use of the slide rule will be allowed for checking but all computations with tables must be shown on the answer paper.

ADVANCED ALGEBRA

See instructions for group II on page 1.

Group II

Answer five questions from this group. Full credit will not be granted unless all operations (except mental ones) necessary to find results are given; simply indicating the operations is not sufficient. Each answer should be reduced to its simplest form. Purely arithmetical solutions for problems will not be accepted.

21 Find, correct to the nearest tenth, the real root of the equation  $x^3 - x - 33 = 0$  [10]

22 One root of the equation  $x^4 - 4x^3 + 6x^2 - 4x + 5 = 0$  is  $2 - i$ ; find the other three roots. [10]

23 Prove that if  $\frac{p}{q}$ , a fraction in its lowest terms, is a root of the equation  $ax^3 + bx + c = 0$ , in which  $a$ ,  $b$  and  $c$  are integers, then  $q$  is a factor of  $a$ . [10]

[The statement that this is a special case of the more general theorem will not be accepted as a proof.]

24 The frictional resistance  $F$  in pounds per square inch in a certain bearing running at a velocity of  $V$  feet per second, is given by the formula  $F = .036V^n$ . Find, correct to the nearest tenth, the value of  $n$ , if  $V = 314$  when  $F = .615$  [10]

25 At what price should a dealer mark a radio that cost him \$108 in order that he may offer a discount of 10% on the marked price and still make a profit of 25% on the selling price? [10]

26 A number consists of three digits which are in arithmetic progression. The number divided by the sum of its digits is equal to 26. If the number is increased by 198, the digits in the units and hundreds places will be interchanged. Find the number. [10]

27 a On the same set of axes plot the graphs of  $(x - 3)^2 + y^2 = 9$  and  $y^2 = x - 3$  [5, 3]

b From the graphs made in answer to a, determine, correct to the nearest tenth, the values of  $x$  and  $y$  common to the two equations. [2]

\*28 Prove that of all rectangles which have a constant perimeter, the square has the maximum area. [10] [Suggestion: Let  $2S$  represent the perimeter of the rectangle.]

\*29 a Write the complex number  $3 + 4i$  in the polar form, giving the value of the amplitude ( $\theta$ ) correct to the nearest degree. [5]

b Find the amplitude of  $(3 + 4i)^3$  [3]

c Find the modulus of  $\sqrt{3 + 4i}$  [2]

\* This question is based on one of the optional topics in the syllabus.

ADVANCED ALGEBRA

Fill in the following lines:

Name of school.....Name of pupil.....

Detach this sheet and hand it in at the close of the one and one half hour period.

Group I

Answer all questions in this group. Each correct answer will receive  $2\frac{1}{2}$  credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

- 1 What is the name of the curve whose equation is  $x^2 + 2y^2 = 20$ ? 1.....
- 2 What is the greatest number of points in which the graph of a linear function can intersect the graph of a quadratic function? 2.....
- 3 Write  $(1 + 2i)^2$  in the form  $a + bi$  3.....
- 4 Write with positive exponents the first two terms of the expansion  $(\frac{1}{x} + x)^{-1}$  4.....
- 5 Write the equation of the straight line which is parallel to the  $x$  axis and whose  $y$  intercept is 4. 5.....
- 6 Write an equation of the lowest possible degree with real coefficients which has for three of its roots 1,  $-1$  and  $i$ . 6.....
- 7 Transform the equation  $x^3 - 3x + 1 = 0$  into an equation whose roots are numerically equal but opposite in sign to the roots of the given equation. 7.....
- 8 Transform the equation  $x^3 - 2 = 0$  into an equation whose roots are less by 1 than the roots of the given equation. 8.....
- 9 For what value of  $k$  are the roots of the equation  $kx^2 + x + 1 = 0$  equal? 9.....
- 10 Find, correct to the nearest hundredth, the value of  $\sqrt[3]{.0837}$  10.....
- 11 How many complex roots has the equation  $x^{10} + 1 = 0$ ? 11.....
- 12 A repeating (recurring) decimal is (a) a rational number or (b) an irrational number. Which is correct, (a) or (b)? 12.....
- 13 Write as an equation the following statement: The weight  $w$  of a spheric solid varies directly as the cube of its radius  $r$ . 13.....
- 14 Solve the equation  $7x^{-\frac{1}{2}} - 4 = 0$  14.....
- 15 How many even numbers of two different digits can be formed from the digits 3, 4, 5, 6 and 8? 15.....
- 16 A committee of four is to be chosen from a group of five men. What is the probability that a certain one of these men will be among those chosen? 16.....
- 17 In a geometric progression the first term is 1 and the ratio is 2. Express in simplest form the sum  $s$  as a function of the number of terms  $n$ . 17.....

ADVANCED ALGEBRA

- 18 If the complex numbers  $4 + 4i$  and  $3i$  are represented graphically by points  $A$  and  $B$  respectively, and their sum is represented by point  $C$ , how many degrees are there in angle  $ACB$ ? 18.....
- 19 Given the function  $y = 2^x$ ; the graph of this function ( $a$ ) has the  $x$ -axis as its axis of symmetry, ( $b$ ) has the  $y$ -axis as its axis of symmetry, or ( $c$ ) is not symmetric with respect to either axis. Which is correct, ( $a$ ), ( $b$ ) or ( $c$ )? 19.....
- 20 Express  $\log_c a$  as the quotient of two logarithms whose base is  $c$ . 20.....