The University of the State of New York

268TH HIGH SCHOOL EXAMINATION

ADVANCED ALGEBRA

Wednesday, January 20, 1937 — 9.15 a. m. to 12.15 p. m., only

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Instructions

Do not open this sheet until the signal is given.

Group I

This group is to be done first and the maximum time allowed for it is one and one half hours.

If you finish group I before the signal to stop is given you may begin group II. However, it is advisable to look your work over carefully before proceeding, since *no credit will be given any answer in group I which is not correct and in its simplest form.*

When the signal to stop is given at the close of the one and one half hour period, work on group I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

Group II

Write at top of first page of answer paper to group II (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in advanced algebra.

The minimum time requirement is five recitations a week for half a school year after the completion of intermediate algebra.

The use of the slide rule will be allowed for checking but all computations with tables must be shown on the answer paper.

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Fill in the following lines:

Name of school.....Name of pupil.....

Detach this sheet and hand it in at the close of the one and one half hour period.

Group I

Answer all questions in this group. Each correct answer will receive 2½ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1 Express $\frac{2}{1-i}$ in the form $a + bi$ *Ans*.....

2 What is the slope of the line whose equation is $4x + 3y = 12$? *Ans*.....

3 If $f(x) = x^3 + 3x^2 - 2x + 1$, what is the value of $f(-2)$? *Ans*.....

4 Transform the equation $x^3 + 2x^2 - 2x + 1 = 0$ into an equation whose roots are greater by 2 than the roots of the given equation. *Ans*.....

5 Write the equation whose roots are 10 times the roots of $x^3 + 3.44x - 0.678 = 0$ *Ans*.....

6 Transform the equation $2x^3 - 3x - 7 = 0$ into an equation whose roots are numerically equal but opposite in sign to the roots of the given equation. *Ans*.....

Questions 7–10 refer to the equation $3x^6 - x^2 - 6 = 0$

7 What is the sum of the roots? *Ans*.....

8 What is the product of the roots? *Ans*.....

9 What is the number of positive roots? *Ans*.....

10 What is the number of complex imaginary roots? *Ans*.....

11 Write the equation of the fourth degree with rational coefficients, three of whose roots are 1, — 1 and i. *Ans*.....

12 How many three-digit numbers can be formed from the digits 1, 2, 3, 4, 5 if no digit is repeated in any one of the numbers? *Ans*.....

13 From a bag containing 2 black and 3 white balls, 2 balls are drawn at random. What is the probability of drawing one of each color? *Ans*.....

14 In how many points does the graph of $y = x^2$ intersect the graph of $y^2 = x$? *Ans*.....

15 Given $a^x = b$; express x in terms of log a and log b. *Ans*.....

16 Find, correct to the *nearest hundredth*, the real value of $(23.38)^{\frac{1}{3}}$ *Ans*.....

17 The first term of a geometric series is 3, the common ratio is ½ and the last term is $1\frac{1}{2}\frac{1}{8}$; find the number of terms of the series. *Ans*.....

18 Write in simplest form the *second* term of the expansion $(\frac{1}{x} - x)^7$ *Ans*.....

[OVER]

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19 If, for $f(x) = 0$, $f(3)$ and $f(4)$ are both positive, then

a $f(x) = 0$ can never have any real roots between 3 and 4.

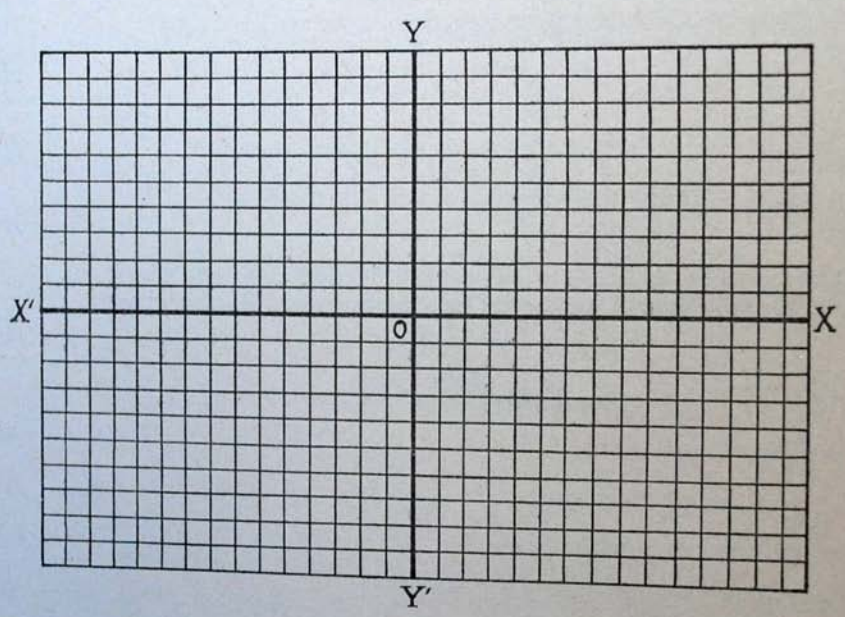
b $f(x) = 0$ may have an even number of real roots between 3 and 4.

c $f(x) = 0$ may have an odd number of real roots between 3 and 4.

Which is correct, *a*, *b* or *c*?

Ans.....

20 On the diagram below add graphically 5 and $3 + 2i$



See instructions for group II on page 1.

Group II

Answer five questions from this group. Full credit will not be granted unless all operations (except mental ones) necessary to find results are given; simply indicating the operations is not sufficient. Each answer should be reduced to its simplest form. Purely arithmetical solutions for problems will not be accepted.

21 Find, correct to the *nearest tenth*, a positive root of

$$x^3 - 6x^2 + 8 = 0 \quad [10]$$

22 Solve completely $6x^4 + 13x^3 + 31x^2 + 3x - 5 = 0$ [10]

23 *a* State the Remainder Theorem. [2]

b Determine k so that $kx^3 - 5x + 2$ divided by $x - 3$ will give a remainder of 5. [4]

c Determine k so that $x - 3$ shall be a factor of $x^3 + kx^2 - 9$ [4]

24 The amount of a sinking fund S created by a fixed investment P, placed annually at compound interest r for n years, is given by the formula $S = \frac{P}{r}[(1 + r)^n - 1]$. With the aid of logarithms compute to the *nearest dollar* the amount of a sinking fund, given $P = \$250$, $n = 15$, $r = 3\frac{1}{2}\%$ [10]

25 An open rectangular tank twice as long as it is wide is to be made from 96 square feet of galvanized iron. If no allowance is made for waste, express the volume of the tank as a function of its width. [10]

26 Two men, C and D, receive different wages. C works 8 days more than D and earns \$168; D earns \$160. If C had received D's wages per day and D had received C's wages, they would have earned together \$16 more. How many days does each work? [6, 4]

27 Solve graphically for the values of x and y common to the following set of equations:

$$y = x^3 - x^2 - 9x + 13$$
$$y = 1 - x \quad [7, 1, 2]$$

*28 Given $y = 2x^3 + 3x^2 - 12x - 10$

a Find the first derivative of y with respect to x. [2]

b Between what values of x is the function decreasing? [2]

c Find the coordinates of the minimum point. [2]

d Find the second derivative of y with respect to x. [2]

e Find the coordinates of the point of inflection. [2]

*29 Express the product $(1 + i\sqrt{3})(-2 - 2i\sqrt{3})$ in polar form. [10]

* This question is based on one of the optional topics in the syllabus.