The University of the State of New York

265TH HIGH SCHOOL EXAMINATION

ADVANCED ALGEBRA

Wednesday, January 22, 1936 — 9.15 a. m. to 12.15 p. m., only

---

Instructions

Do not open this sheet until the signal is given.

Group I

This group is to be done first and the maximum time allowed for it is one and one half hours.

If you finish group I before the signal to stop is given you may begin group II. However, it is advisable to look your work over carefully before proceeding, since *no credit will be given any answer in group I which is not correct and in its simplest form.*

When the signal to stop is given at the close of the one and one half hour period, work on group I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

Group II

Write at top of first page of answer paper to group II (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in advanced algebra.

The minimum time requirement is five recitations a week for half a school year after the completion of intermediate algebra.

The use of the slide rule will be allowed for checking but all computations with tables must be shown on the answer paper.

Fill in the following lines:

Name of school.....Name of pupil.....

Detach this sheet and hand it in at the close of the one and one half hour period.

Group I

Answer all questions in this group. Each correct answer will receive 2½ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1 How many imaginary roots has the equation $x^6 + 3x^2 + 4 = 0$? *Ans*.....

2 What is the sum of the roots of the equation
$2x^3 + 3x^2 - 6x + k = 0$? *Ans*.....

3 Write in the form $y = mx + b$ the equation of the straight line whose y-intercept is 3 and which is parallel to the straight line whose equation is $y = 2x + 4$ *Ans*.....

4 Given $y = \log_2 x$; does y increase or decrease as x increases? *Ans*.....

5 Is the repeating decimal 2.7373... a rational or an irrational number? *Ans*.....

6 Express in the form $a + bi$ the result of dividing $2i + 2$ by i^7 *Ans*.....

7 Find, correct to the *nearest hundredth*, the real root of the equation $x^5 = 578$ *Ans*.....

8 Find the negative integral root of the equation
$2x^4 + 5x^3 - 4x^2 - 4x - 3 = 0$ *Ans*.....

9 For what rational value of x does $x^{-\frac{3}{2}}$ equal 8? *Ans*.....

10 Transform the equation $3x^3 - 2x + 1 = 0$ into an equation with integral coefficients, that of the highest-degree term being unity. *Ans*.....

11 Transform the equation $x^3 - x^2 - 6x - 4 = 0$ into an equation whose roots are greater by 1 than the roots of the given equation. *Ans*.....

12 From a group of 4 men and 3 women a committee of three is chosen. What is the probability that the committee will consist of three men? *Ans*.....

13 How many integers are there between 100 and 1000 in which no digit is repeated? *Ans*.....

14 Write the first *two* terms of the expansion $(x - \frac{1}{x})^8$ *Ans*.....

Indicate whether each of the statements 15 to 19 is *true* or *false* by writing the word *true* or *false* on the dotted line at the right.

15 A rational integral equation with real coefficients may have an odd number of imaginary roots. *Ans*.....

16 If x, y and z are in geometric progression, then $\log x$, $\log y$ and $\log z$ are in arithmetic progression. *Ans*.....

17 The graph of $y = 2^x$ is symmetric with respect to the y-axis. *Ans*.....

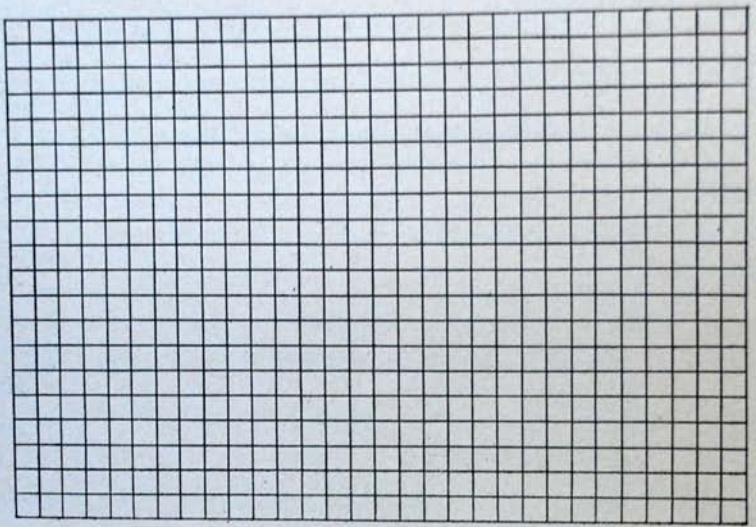
18 The graph of $y = x^2 + kx + 9$ is tangent to the x-axis for two and only two values of k. *Ans*.....

[OVER]

581

19 If the rational integral function $a_0x^5 + a_1x^4 + \ldots + a_4x + a_5$, whose coefficients are real numbers, has 3 changes of sign, its graph will always cross the x-axis 3 times at points whose abscissas are positive. *Ans.*.....

20 On the diagram below represent graphically any two complex conjugate numbers and find their sum graphically.



190

See instructions for group II on page 1.

Group II

Answer five questions from this group. Full credit will not be granted unless all operations (except mental ones) necessary to find results are given; simply indicating the operations is not sufficient. Each answer should be reduced to its simplest form. Purely arithmetical solutions for problems will not be accepted.

21 The equation $x^3 - 5x^2 + 4x + 5 = 0$ has a real root that lies between 2 and 3. Find this root correct to the *nearest hundredth*. [10]

22 The equation $x^3 + kx + 16 = 0$ has two equal roots; find the real value of k. [10]

23 Given the function $y = 20\ e^{rx}$, the value of e being 2.718; find, correct to the *nearest tenth*, the value of r if $y = 200$ when $x = \frac{1}{2}$ [10]

24 Assuming p and q to be real numbers, prove that if $a + bi$ is a root of $x^2 + px + q = 0$, then $a - bi$ is also a root. [10]
[The statement that this is a particular case of the general theorem will not be accepted as a proof.]

25 A diagonal of a rectangle is 2 inches and its area is a square inches. Express a side of the rectangle as a function of a and show that the solution is admissible only when a equals or is less than 2 square inches. [7, 3]

26 A motorist after traveling for an hour found it necessary to reduce his rate to 30 miles per hour and as a result arrived at his destination 25 minutes late. If he had traveled the entire journey at 30 miles per hour, he would have arrived 45 minutes late. How fast was he traveling during the first hour? [10]

27 *a* Draw the graph of the equation $x^2 + xy = 14$ from $x = +1$ to $x = +7$ inclusive. [7]
b Using the same set of axes as in *a*, draw the graph of the equation $y = x$ [1]
c From the graphs made in answer to *a* and *b* determine, correct to the *nearest tenth*, a solution common to both equations. [2]

*28 Differentiate *each* of the following functions:
a $y = x^3 - 2x + 3$ [2]
b $y = \sqrt{2x^2 - 5x}$ [4]
c $y = \dfrac{x^2 + 1}{x - 1}$ [4]

*29 In a certain college preparatory school 102 pupils were examined and found to have mental ages as given in the following table:

Mental age in years	12	13	14	15	16	17	18	19	20
Number of pupils	1	6	9	12	24	28	14	6	2

a From these data determine in which mental-age year (1) the median lies, (2) the mode lies. [3, 1]
b Represent these data graphically. [3]
c Is the distribution symmetrical or nonsymmetrical; that is, does it follow the normal probability? In general, what does the distribution show concerning the mental ability of this group of pupils? [2, 1]

* This question is based on one of the optional topics in the syllabus.