The University of the State of New York

238TH HIGH SCHOOL EXAMINATION

ADVANCED ALGEBRA

Thursday, January 20, 1927 — 9.15 a. m. to 12.15 p. m., only

Write at top of first page of answer paper (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in (1) elementary algebra, (2) intermediate algebra, (3) advanced algebra. The minimum time requirement is five recitations a week in algebra for two school years.

Answer eight questions. Each answer should be reduced to its simplest form.

In the examination in advanced algebra the use of the slide rule will be allowed for checking, provided all computations with tables are shown on the answer paper.

1 *a* Write in factor form an equation of the fourth degree in x having one positive fractional root, one negative integral root and one pair of conjugate complex roots. [6½]

b Write an equation of the fifth degree in x having both the sum and the product of its roots equal to zero. [6]

2 *a* Write the equation whose roots are the roots of the equation $x^3 - 20x^2 + 500x - 4000 = 0$ divided by 10. [Solution of equation not required] [6]

b Transform the equation $36x^3 + 18x^2 + 2x + 1 = 0$ into an equation that can not have a rational fractional root. [Solution of equation not required] [6½]

3 *a* Transform the equation $2x^2 - 6x + 5 = 0$ into an equation lacking the second term. [6]

b Solve the transformed equation obtained in answer to *a* and from the roots thus found determine the roots of the given equation. [6½]

4 *a* Without solving determine the nature of the roots of the equation $2x^6 + 3x^4 - 2x^2 - 5 = 0$. [6½]

b Determine m and c so that the graph of the equation $y = mx + c$ shall pass through the points (2, 5) and (1, 0). [6]

5 Find to the *nearest hundredth* the positive root of $x^3 - x^2 - 7 = 0$ [12½]

6 The sum of an infinite number of terms of a geometric series is 12 and the sum of their square roots is 6. Write the first three terms of the series. [12½]

7 *a* How many different passenger tickets will a railway company need for use on a division on which there are 20 stations? [6½]

b How many *odd* numbers, each having 5 digits, can be formed with the digits 1, 2, 3, 4, 5, if no repetition of a digit is allowed? [6]

8 *a* If $y = 2^n$, by what number is y multiplied when n is increased by 3? [3½]

b If $c = \sqrt{a^2 + b^2}$, by what number is c multiplied when a and b are both tripled? [3]

c If $x = \dfrac{a}{b - \frac{c}{d}}$, all the letters being positive, and d is increased while the other letters remain constant,

(1) then $\frac{c}{d}$ (increases, decreases)

(2) then $b - \frac{c}{d}$ (increases, decreases)

(3) then x (increases, decreases)

Copy *each* of the last three statements, omitting the incorrect word. [6]

9 *a* In what period of time will a sum of money double itself at 5% compound interest, the interest being compounded semiannually? [6½]

b Show that a sum of money will increase more than a hundredfold in 95 years at 5% compound interest, the interest being compounded annually. [6]

10 *a* Prove that, if the corresponding terms of two arithmetic series are added, the result will also be an arithmetic series. [4]

b Find by the use of logarithms the sum of the series 1, 1.05, $(1.05)^2$, . . . to 20 terms. [8½]

11 *a* The radius of a circle is 4 and its center is at the origin. Write *two* conjugate complex numbers located on the circle if the real part of each is -2. Write *two* conjugate complex numbers located on the circle if the imaginary part of each is $2i\sqrt{3}$. [6]

b Expand $(a + b)^{-4}$ to *four* terms by the binomial theorem. [6½]

12 For what values of k will the graph of $y = x^3 - 3x + k$ be tangent to the x-axis? [The drawing of the graph is not required.] [12½]

13 A rectangular box is open at the top and has a square base. Its volume is 18 cubic inches and its outside surface 33 square inches. Find the dimensions of the box. [6½, 6]

14 In the equation $y = \frac{x^3}{50} + \frac{135}{x}$, y is the cost in dollars per mile to run a certain steamer at the rate of x miles per hour. Make a graph of the equation for values of x from 5 to 25 inclusive at intervals of 5. From the graph determine the least value of y, that is, the most economical rate at which to run the steamer. [9, 3½]