New York State Education Department

206TH HIGH SCHOOL EXAMINATION

ADVANCED ALGEBRA

Monday, January 15, 1912—9.15 a. m. to 12.15 p. m., only

Write at top of first page of answer paper (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in algebra.

The minimum time requirement is five recitations a week in algebra for two school years.

Answer eight questions. No credit will be allowed unless all operations (except mental ones) necessary to find results are given; simply indicating the operations is not sufficient.

1 Solve $2\sqrt{x^2-2x-3}+x^2-2x=6$

2 Prove that the number of combinations of n things taken r at a time is equal to the number of combinations of n things taken $n-r$ at a time.

If the number of permutations of n things taken 3 at a time is equal to 10 times the number of permutations of n things taken 2 at a time, what is the value of n?

3 Represent graphically the complex numbers

$$-3+2\sqrt{-1} \text{ and } 1-4\sqrt{-1};$$

then represent their sum and their difference.

4 Prove that if a determinant has two rows identical, its value is zero.

Evaluate the determinant $\begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 1+x & 1 & 1 \\ 1 & 1 & 1+x & 1 \\ 1 & 1 & 1 & 1+x \end{vmatrix}$

5 Find by means of determinants the value of y from the following equations:

$$\begin{aligned} 3x-7y+5z&=9 \\ x+8y-3z&=4 \\ 5x-4z-2&=0 \end{aligned}$$

6 Prove that if $a+b\sqrt{-1}$ is a root of an equation with real coefficients $a-b\sqrt{-1}$ is also a root.

7 Transform $x^3-\frac{x^2}{2}+\frac{15x}{36}-\frac{13}{108}=0$ into an equation whose coefficients are integers, the coefficient of the first term being 1. Find a rational root of the resulting equation. What is the commensurable root of the original equation?

8 Apply Descartes' rule of signs to determine the nature of the roots of the equation $x^3+3x-5=0$

Graph the function $y=x^3+3x-5$. What information concerning the roots of the equation is afforded by the graph?

9 One root of the equation $x^4-4x^3+5x^2+2x+52=0$ is $3-2\sqrt{-1}$; find the other roots.

10 In the equation $2x^3+3x-90=0$, find by Horner's method, to *two* decimal places, a positive root lying between 3 and 4.