ADVANCED ALGEBRA

Tuesday, January 24, 1905—9.15 a. m. to 12.15 p. m., only

*Answer eight questions but **no more**. If more than eight are answered only the first eight answers will be considered. Give each step of solution. Reduce fractions to lowest terms. Express final result in its simplest form and mark it Ans. Each complete answer will receive 12½ credits. Papers entitled to 75 or more credits will be accepted.*

1 Discuss the divisibility of $x^n \pm y^n$ by $x \pm y$

2 Which is the greater $\frac{a+b}{2}$ or $\frac{2ab}{a+b}$, a and b being unequal positive integers?

3 If $a + b\sqrt{-1} = c + d\sqrt{-1}$, prove that $a = c$ and $b = d$

4 Write the 10th term of $(\frac{1}{3}x^{-2} - 2y^{-3})^{\frac{1}{4}}$

5 If a stack of hay will last a horse c months and a cow d months, how many months will it last a horses and b cows feeding together?

6 $\sqrt[3]{5+x} + \sqrt[3]{5-x} = \sqrt[3]{10}$ is an equation; cubing and simplifying . . . $\sqrt[3]{(5+x)^2}\sqrt[3]{5-x} + \sqrt[3]{5+x}\sqrt[3]{(5-x)^2} = 0$; dividing by $\sqrt[3]{5+x}\sqrt[3]{5-x}$ . . . $\sqrt[3]{5+x} + \sqrt[3]{5-x} = 0$; whence $+5 = -5$. Discuss the error that led to this absurdity and find *two* roots of the equation.

7 In an arithmetic series, derive the value of S in terms of a, l and d.

8 Find the cube root of

$$\frac{1}{y^{-6}} - 12y^5 + \frac{195}{4y^{-4}} - 70y^3 + \frac{195}{16y^{-2}} - \frac{3y}{4} + 64^{-1}$$

9 State *three* properties or principles of logarithms. Prove *one* of them.

10 Develop $\frac{2}{3x - 2x^2}$ into a series of *five* terms, by the method of undetermined coefficients.

11 Solve $x^5 - 3x^4 - 5x^3 + 15x^2 + 4x = 12$

12 Transform $\frac{1}{3}x^4 - \frac{1}{4}x^3 - \frac{1}{3}x^2 + \frac{2}{3} = 0$ into an equation with integral coefficients, that of the first term being unity. How do the roots of the resulting equation compare in value with those of the original equation?