High School Department

176TH EXAMINATION

ADVANCED ALGEBRA

Tuesday, January 27, 1903—9.15 a. m. to 12.15 p. m., only

Answer eight questions but **no more.** *If more than eight are answered only the first eight answers will be considered. Give each step of solution. Reduce fractions to lowest terms. Express final result in its simplest form and mark it Ans. Each complete answer will receive 12½ credits. Papers entitled to 75 or more credits will be accepted.*

1 Simplify

$$\left(x^{-a}\right)^{b}\div\left(x^{-a}\right)^{-b}+\left[\left(x^{1-\frac{a}{b}}\right)^{\frac{1}{a-b}}\times\sqrt[b]{\left(x^{-\frac{1}{a}}\right)^{a}}\right]^{2}\div\left(\frac{1}{x^{\frac{2}{a}}}\right)^{\frac{a}{b}}$$

2 Expand to *four* terms by the binomial formula $\left(x^{-1}+3y^{\frac{1}{2}}\right)^{-\frac{1}{6}}$, giving all the work for finding the coefficients.

3 By the method of synthetic division find the quotient of $a^7-2a^6b+a^5b^2+2a^3b^4+3a^2b^5-3ab^6$ divided by $a^3+ab^2-b^3$

4 Solve $\begin{cases} x^y=y^x \\ x^3-y^2=0 \end{cases}$

5 Prove that a quadratic equation can have but two roots.

6 Resolve into partial fractions $\frac{3x^3-8x^2+9}{(x-1)^4}$

7 Determine by a general method the equal roots of

$$x^4-10x^3+37x^2-60x+36=0$$

8 Prove that in an equation with real coefficients imaginary roots occur in pairs, if at all.

9 Find *one* limit of each of two numbers if twice the first plus the second number equals 16, and the first plus the second is greater than 13 minus half the second.

10 Derive a formula for finding (*a*) the sum of the terms of an arithmetic progression, (*b*) the harmonic mean between two given quantities.

11 Given $\sqrt{5}=2.236$, $\sqrt{6}=2.449$, $\sqrt{7}=2.645$, $\sqrt{8}=2.828$; find by interpolation $\sqrt{5.8}$

12 Explain and illustrate (*a*) *four* applications of logarithms to arithmetic computations, (*b*) *two* advantages of the common system over the Napierian.