G.C.B.5: Arc Length 1

1. The diagram below shows circle $O$ with radii $OA$ and $OB$. The measure of angle $AOB$ is $120^\circ$, and the length of a radius is 6 inches.

Which expression represents the length of arc $AB$, in inches?

1) $\frac{120}{360} (6\pi)$
2) $120(6)$
3) $\frac{1}{3} (36\pi)$
4) $\frac{1}{3} (12\pi)$

2. A circle is drawn to represent a pizza with a 12 inch diameter. The circle is cut into eight congruent pieces. What is the length of the outer edge of any one piece of this circle?

1) $\frac{3\pi}{4}$
2) $\pi$
3) $\frac{3\pi}{2}$
4) $3\pi$

3. A regular hexagon is inscribed in a circle. What is the ratio of the length of a side of the hexagon to the minor arc that it intercepts?

1) $\frac{\pi}{6}$
2) $\frac{3}{6}$
3) $\frac{3}{\pi}$
4) $\frac{6}{\pi}$

4. An electron travels along a circular path with a radius of 4.6 miles. What is the number of miles the electron traveled during an interval when the central angle formed by the electron’s path was $220^\circ$?

1) 3.84
2) 8.83
3) 17.66
4) 1012

5. In the diagram below, two concentric circles with center $O$, and radii $OC$, $OD$, $OGE$, and $ODF$ are drawn.

If $OC = 4$ and $OE = 6$, which relationship between the length of arc $EF$ and the length of arc $CD$ is always true?

1) The length of arc $EF$ is 2 units longer than the length of arc $CD$.
2) The length of arc $EF$ is 4 units longer than the length of arc $CD$.
3) The length of arc $EF$ is 1.5 times the length of arc $CD$.
4) The length of arc $EF$ is 2.0 times the length of arc $CD$. 
6 A ball is rolling in a circular path that has a radius of 10 inches, as shown in the accompanying diagram. What distance has the ball rolled when the subtended arc is 54°? Express your answer to the nearest hundredth of an inch.

7 The accompanying diagram shows the path of a cart traveling on a circular track of radius 2.40 meters. The cart starts at point $A$ and stops at point $B$, moving in a counterclockwise direction. What is the length of minor arc $AB$, over which the cart traveled, to the nearest tenth of a meter?

8 Cities $H$ and $K$ are located on the same line of longitude and the difference in the latitude of these cities is 9°, as shown in the accompanying diagram. If Earth’s radius is 3,954 miles, how many miles north of city $K$ is city $H$ along arc $HK$? Round your answer to the nearest tenth of a mile.

9 Circle $O$ shown below has a radius of 12 centimeters. To the nearest tenth of a centimeter, determine the length of the arc, $x$, subtended by an angle of $83°50'$.

10 A sprinkler system is set up to water the sector shown in the accompanying diagram, with angle $ABC$ measuring 1 radian and radius $AB = 20$ feet. What is the length of arc $AC$, in feet?

11 A circle has a radius of 4 inches. In inches, what is the length of the arc intercepted by a central angle of 2 radians?

12 Jack wants to plant a border of flowers in the shape of an arc along the edge of a circular walkway. If the circle has a radius of 5 yards and the angle subtended by the arc measures $1 \frac{1}{2}$ radians, what is the length, in yards, of the border?
13. In a circle with a diameter of 24 cm, a central angle of \(\frac{4\pi}{3}\) radians intercepts an arc. The length of the arc, in centimeters, is
1) \(8\pi\)
2) \(9\pi\)
3) \(16\pi\)
4) \(32\pi\)

14. A wheel has a radius of 18 inches. Which distance, to the nearest inch, does the wheel travel when it rotates through an angle of \(\frac{2\pi}{5}\) radians?
1) 45
2) 23
3) 13
4) 11

15. A circle has a radius of 12 units. For this circle, which expression incorrectly states the length of the arc intercepted by the given central angle?
1) angle = 120°, arc length = \(8\pi\)
2) angle = 6°, arc length = 72
3) angle = \(\frac{2}{3}\) radian, arc length = 8
4) angle = \(\frac{\pi}{3}\) radians, arc length = \(4\pi\)

16. In a circle whose radius is 10, what is the length of the arc intercepted by a central angle of 4 radians?

17. In a circle with a radius of 3 centimeters, find, in centimeters, the length of an arc intercepted by a central angle of 2 radians.

18. In a circle of radius 8, find the length of the arc intercepted by a central angle of 1.5 radians.

19. Circle \(O\) has a radius of 10. Find the length of an arc subtended by a central angle measuring 1.5 radians.

20. Express, in terms of \(\pi\), the length of the arc intercepted by a central angle of \(\frac{\pi}{6}\) radian in a circle with radius 30.

21. In the diagram below, Circle 1 has radius 4, while Circle 2 has radius 6.5. Angle \(A\) intercepts an arc of length \(\pi\), and angle \(B\) intercepts an arc of length \(\frac{13\pi}{8}\).

Dominic thinks that angles \(A\) and \(B\) have the same radian measure. State whether Dominic is correct or not. Explain why.
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Answer Section

1 ANS: 4

\[ C = 12\pi \frac{120}{360} (12\pi) = \frac{1}{3} (12\pi) \]

REF: 061822geo

2 ANS: 3

\[ s = \theta r = \frac{2\pi}{8} \cdot 6 = \frac{3\pi}{2} \]

REF: 061212a2

3 ANS: 3

Assume the circle has a radius of 1. The hexagon can be divided into six equal equilateral triangles with sides of 1. The side of the hexagon is 1. The central angle created by the two legs of the triangle, \( \theta \), is 60° or \( \frac{\pi}{3} \) radians. \( s = \theta r = \frac{\pi}{3} \cdot 1 = \frac{\pi}{3} \). The ratio of the length of a side of the hexagon to the minor arc that it intercepts is \( \frac{1}{\pi} \) or \( \frac{3}{\pi} \).

REF: 080109b

4 ANS: 3

\[ s = \theta r = 220 \left( \frac{\pi}{180} \right) \cdot 4.6 \approx 17.66 \]

REF: 081622a2

5 ANS: 3

\[ \frac{s_L}{s_S} = \frac{6\theta}{4\theta} = 1.5 \]

REF: 011824geo

6 ANS:

9.42. \( \frac{\pi}{180} = \frac{3\pi}{10} \) radians. \( s = \theta r = \frac{3\pi}{10} \cdot 10 = 3\pi \approx 9.42 \).

REF: 010223b
7 ANS: 
\[
165 \frac{\pi}{180} = \frac{11\pi}{12} \text{ radians. } s = \theta r = \frac{11\pi}{12} \cdot 2.4 \approx 6.9.
\]
REF: 080524b

8 ANS: 
\[
621.1. \ 9 \cdot \frac{\pi}{180} = \frac{\pi}{20} \text{ radians. } s = \theta r = \frac{\pi}{20} \cdot 3954 \approx 621.1.
\]
REF: 080426b

9 ANS: 
\[
83^\circ 50' \cdot \frac{\pi}{180} \approx 1.463 \text{ radians } s = \theta r = 1.463 \cdot 12 \approx 17.6
\]
REF: 011435a2

10 ANS: 3 
\[
s = \theta r = 1 \cdot 20 = 20
\]
REF: 060818b

11 ANS: 4 
\[
s = \theta r = 2 \cdot 4 = 8
\]
REF: fall0922a2

12 ANS: 4 
\[
s = \theta r = 1 \frac{1}{2} \cdot 5 = 7.5.
\]
REF: 010806b

13 ANS: 3 
\[
s = \theta r = \frac{4\pi}{3} \cdot \frac{24}{2} = 16\pi
\]
REF: 011611a2

14 ANS: 2 
\[
s = \theta r = \frac{2\pi}{5} \cdot 18 \approx 23
\]
REF: 011526a2

15 ANS: 2 
\[
\frac{72}{6 \cdot \frac{\pi}{180}} \neq 12
\]
REF: 011722a2
16 ANS: 
\[ s = \theta r = 4 \cdot 10 = 40 \]

REF: 010415siii

17 ANS: 
\[ s = \theta r = 2 \cdot 3 = 6 \]

REF: 068514siii

18 ANS: 
\[ s = \theta r = 1.5 \cdot 8 = 12 \]

REF: 068713siii

19 ANS: 
\[ s = \theta r = 1.5 \cdot 10 = 15 \]

REF: 069714siii

20 ANS: 
\[ s = \theta r = \frac{\pi}{6} \cdot 30 = 5\pi \]

REF: 089313siii

21 ANS: 
\[ s = \theta \cdot r \quad s = \theta \cdot r \quad \text{Yes, both angles are equal.} \]

\[ \pi = A \cdot 4 \quad \frac{13\pi}{8} = B \cdot 6.5 \]

\[ \frac{\pi}{4} = A \quad \frac{\pi}{4} = B \]

REF: 061629geo